REINFORCEMENT LEARNING IN CONTINUING PROBLEMS USING AVERAGE REWARD

Defense Talk
28 March 2024

Abhishek Naik

with thanks to Rich, Yi, Janey, and many others

15th Aug 2019

MY GOAL

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To develop simple and practical learning algorithms
from first principles for long-lived agents

TOPICS I WORKED ON DURING MY PH.D.

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1. *One-step* average-reward methods

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1. *One-step* average-reward methods
2. *Multi-step* average-reward methods

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1. *One-step* average-reward methods
2. *Multi-step* average-reward methods
3. An idea to improve *discounted-reward* methods

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4. A suite of continuing problems

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- Generalizing in the action space for large recommender systems

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Problem setting

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Conclusions, limitations, and future work

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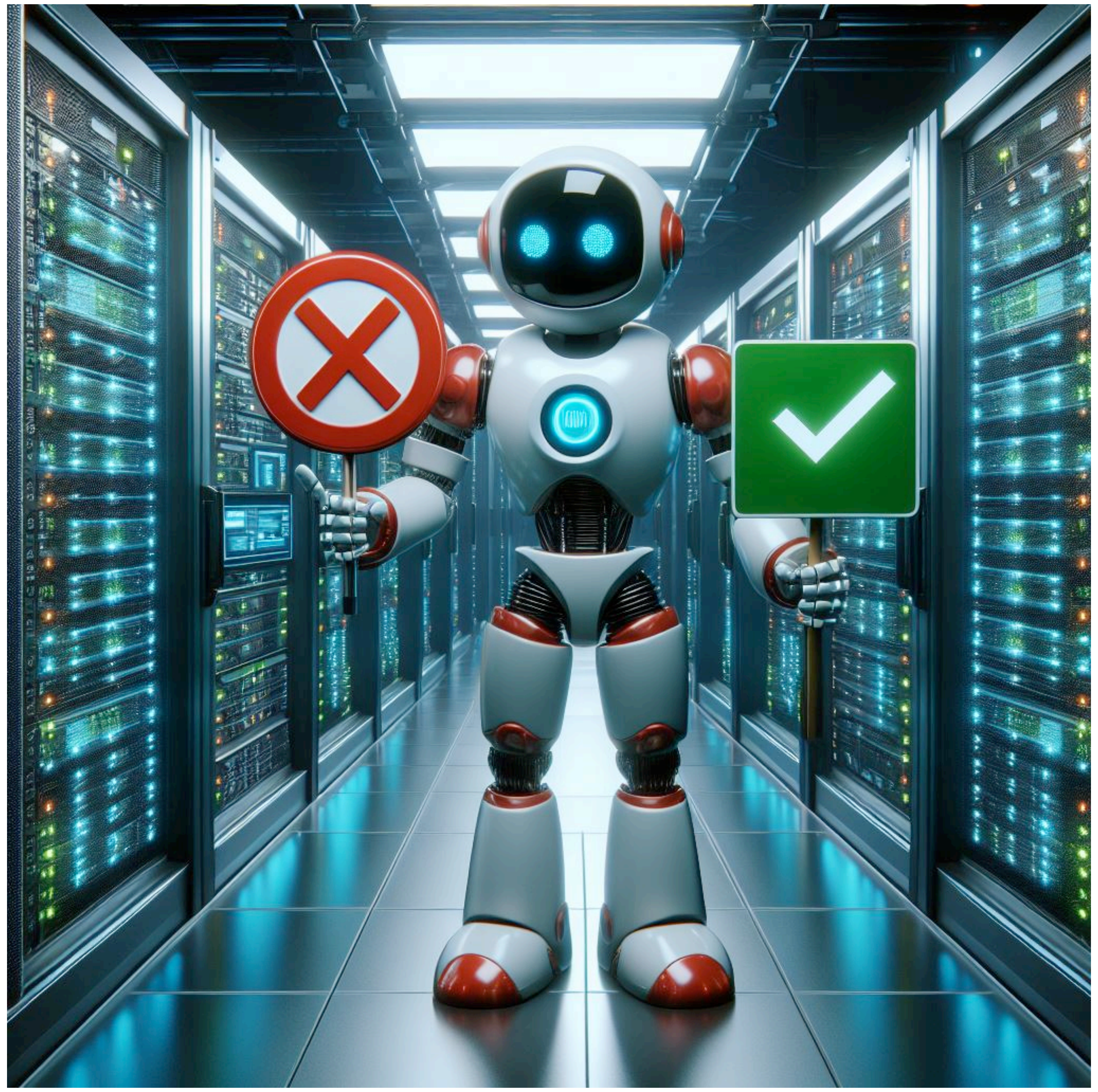

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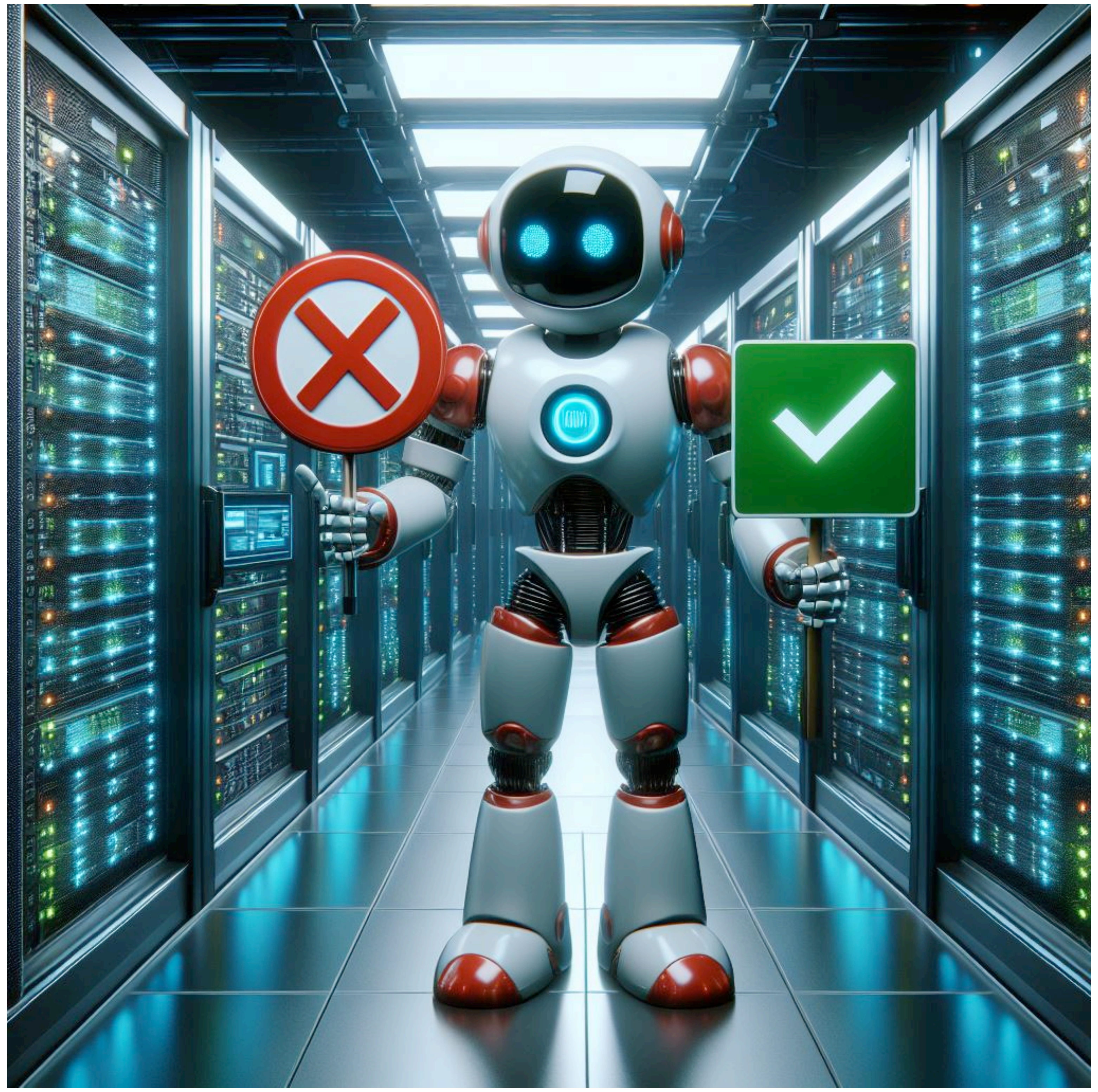

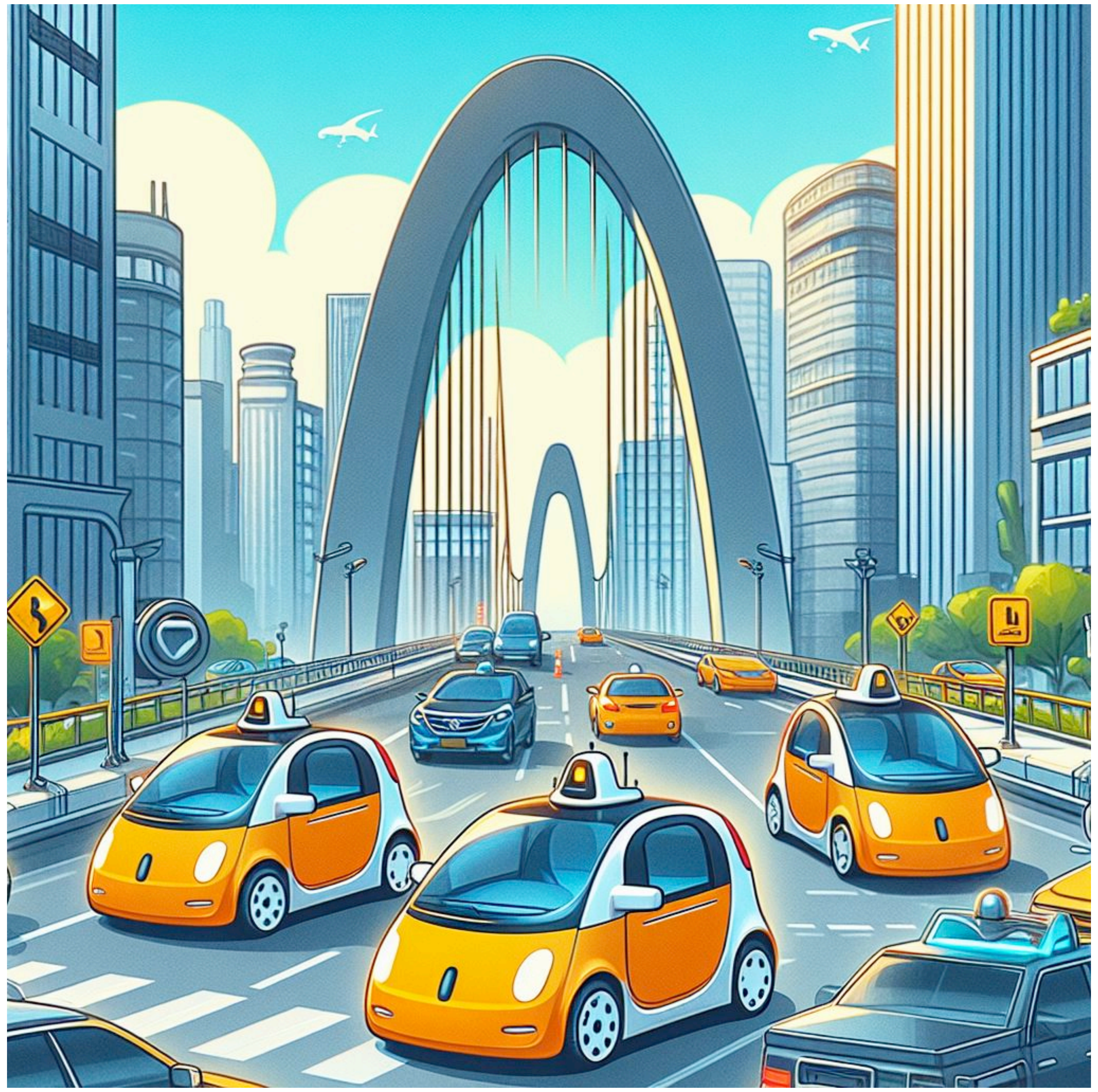

Images generated using DALL·E 3

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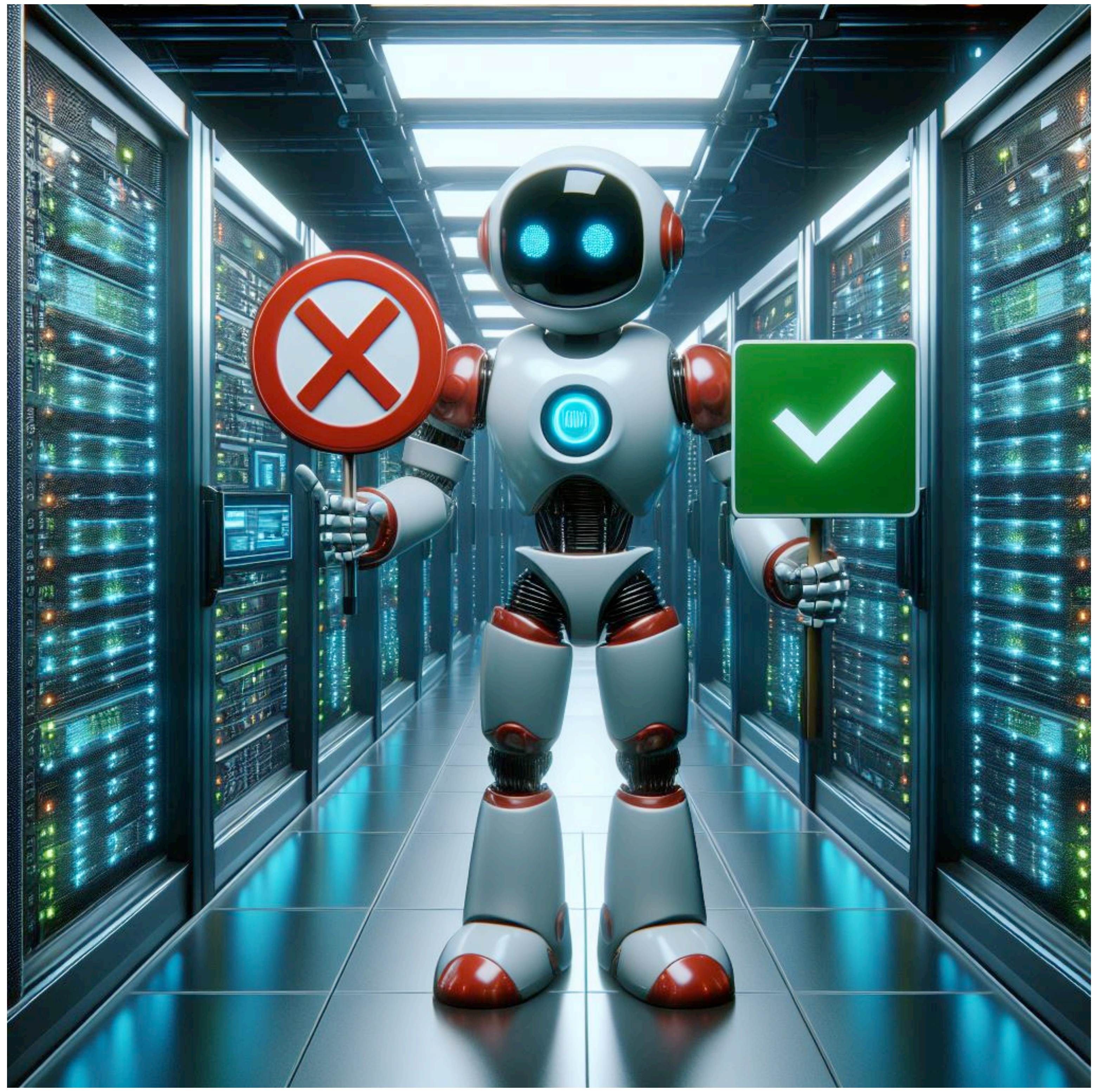

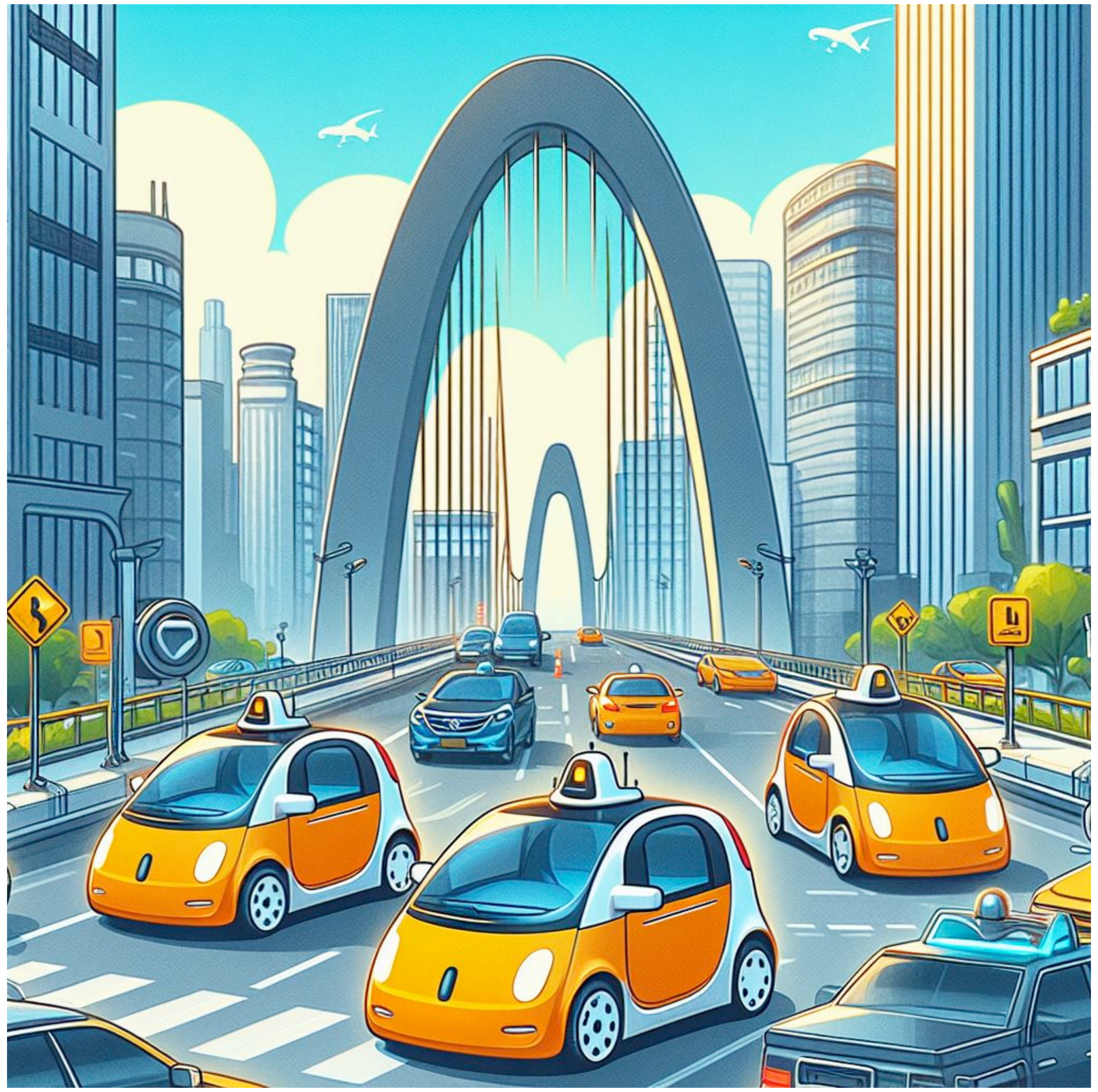

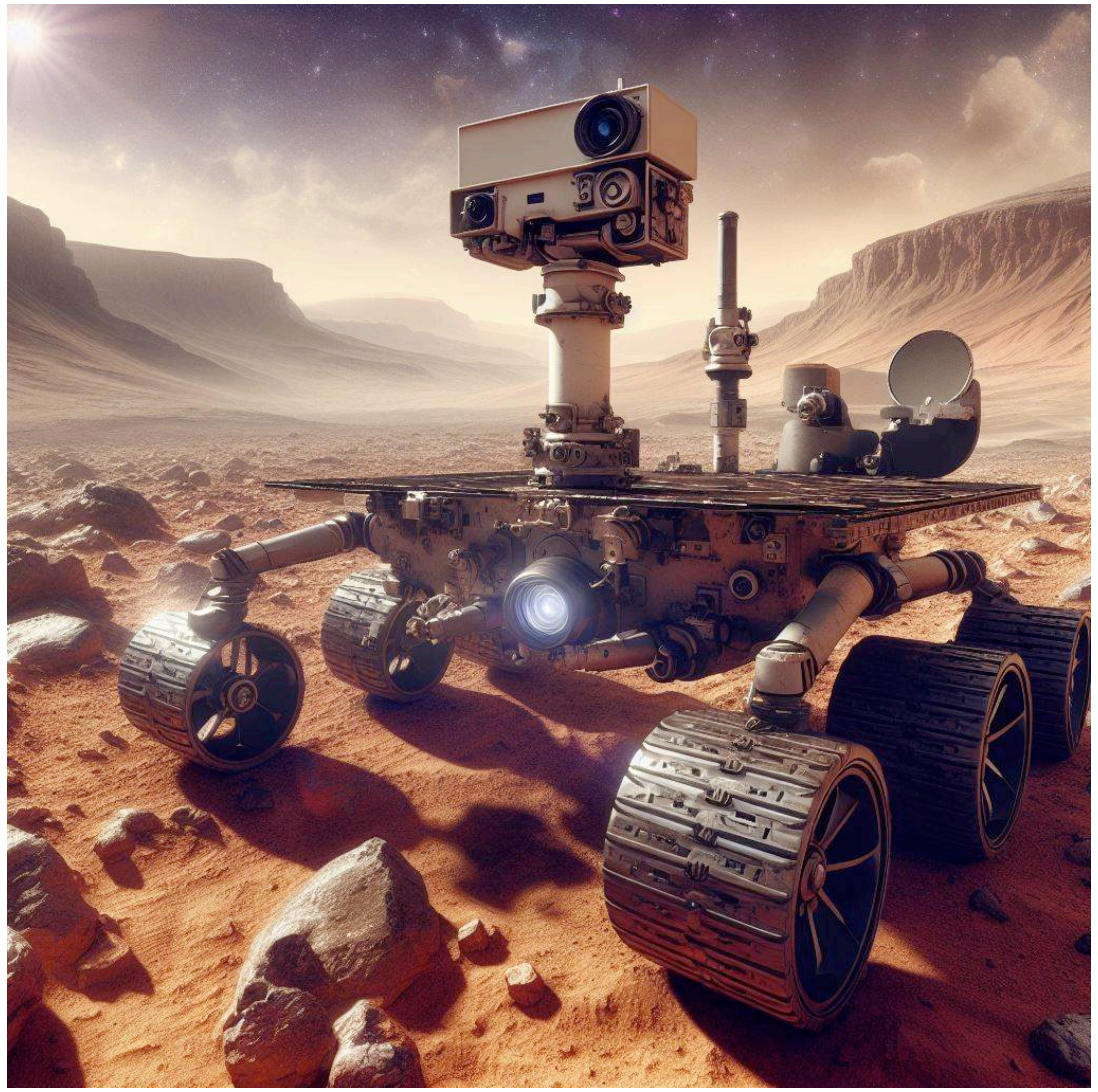

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CONTINUING PROBLEMS: FORMULATIONS

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$$S_0\ A_0\ R_1\ S_1\ A_1, R_2 \ldots\ S_t\ A_t\ R_{t+1}\ S_{t+1}\ A_{t+1}\ R_{t+2}\ \ldots$$

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$$\max_{\pi} \sum_{t}^{\infty} R_t$$

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$$r(\pi) \doteq \lim_{n \to \infty} \frac{1}{n} \mathbb{E}_{\pi}\Big[\sum_{t=1}^{n} R_t\Big]$$

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Average-Reward Formulation

$$\max_{\pi} \; r(\pi)$$

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Estimate $r(\pi)$ and $\tilde{v}_\pi$

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Find π that maximizes $r(\pi)$

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The Control Problem

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The Prediction Problem

Estimate $r(\pi)$ and $\tilde{v}_\pi$

while behaving according to b

The Control Problem

Find π that maximizes $r(\pi)$

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The Prediction Problem

Estimate $r(\pi)$ and $\tilde{v}_\pi$

while behaving according to b

The Control Problem

Find π that maximizes $r(\pi)$

while behaving according to b

off-policy

OUTLINE

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Conclusions, limitations, and future work

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ONE-STEP AVERAGE-REWARD LEARNING METHODS

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- Abounadi, Bertsekas, & Borkar (2001): *a big step forward*

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ESTIMATING THE AVERAGE REWARD FROM DATA

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ESTIMATING THE VALUES FROM DATA

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$$q^\gamma_\pi(s,a) \doteq \mathbb{E}_\pi[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots \mid S_t = s, A_t = a]$$

$$q^\gamma_*(s,a) = \sum_{s',r} p(s',r \mid s,a)\left[r + \gamma \max_{a'} q^\gamma_*(s',a')\right]$$

$$\tilde{q}_\pi(s,a) \doteq \mathbb{E}_\pi[R_{t+1} - r(\pi) + R_{t+2} - r(\pi) + \ldots \mid S_t = s, A_t = a]$$

$$\tilde{q}_*(s,a) = \sum_{s',r} p(s',r \mid s,a)\left[r - \bar{r} + \max_{a'} \tilde{q}_*(s',a')\right]$$

Discounted Q-learning

$$Q_{t+1}(S_t,A_t) \doteq Q_t(S_t,A_t) + \alpha_t\underbrace{\left[R_{t+1} + \gamma \max_{a'} Q_t(S_{t+1},a') - Q_t(S_t,A_t)\right]}_{\delta^\gamma_t}$$

$$Q_{t+1}(S_t,A_t) \doteq Q_t(S_t,A_t) + \alpha_t\left[R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1},a') - Q_t(S_t,A_t)\right]$$

new_estimate = old_estimate + stepsize * (new_target - old_estimate)

ESTIMATING THE VALUES FROM DATA

$$S_0 \; A_0 \; R_1 \; S_1 \; A_1, R_2 \dots \; S_t \; A_t \; R_{t+1} \; S_{t+1} \; A_{t+1} \; R_{t+2} \; \dots$$

$$q^\gamma_\pi(s,a) \doteq \mathbb{E}_\pi[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots \mid S_t = s, A_t = a]$$

$$q^\gamma_*(s,a) = \sum_{s',r} p(s',r \mid s,a)\big[r + \gamma \max_{a'} q^\gamma_*(s',a')\big]$$

$$\tilde{q}_\pi(s,a) \doteq \mathbb{E}_\pi[R_{t+1} - r(\pi) + R_{t+2} - r(\pi) + \dots \mid S_t = s, A_t = a]$$

$$\tilde{q}_*(s,a) = \sum_{s',r} p(s',r \mid s,a)\big[r - \bar{r} + \max_{a'} \tilde{q}_*(s',a')\big]$$

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$$Q_{t+1}(S_t,A_t) \doteq Q_t(S_t,A_t) + \alpha_t\underbrace{\big[R_{t+1} + \gamma \max_{a'} Q_t(S_{t+1},a') - Q_t(S_t,A_t)\big]}_{\delta^\gamma_t}$$

$$Q_{t+1}(S_t,A_t) \doteq Q_t(S_t,A_t) + \alpha_t\big[R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1},a') - Q_t(S_t,A_t)\big]$$

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ESTIMATING THE VALUES FROM DATA

$$S_0\ A_0\ R_1\ S_1\ A_1, R_2 \ldots\ S_t\ A_t\ R_{t+1}\ S_{t+1}\ A_{t+1}\ R_{t+2}\ \ldots$$

$$q^\gamma_\pi(s,a) \doteq \mathbb{E}_\pi[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots \,|\, S_t = s, A_t = a]$$

$$q^\gamma_*(s,a) = \sum_{s',r} p(s',r\,|\,s,a)\left[r + \gamma \max_{a'} q^\gamma_*(s',a')\right]$$

$$\tilde{q}_\pi(s,a) \doteq \mathbb{E}_\pi[R_{t+1} - r(\pi) + R_{t+2} - r(\pi) + \ldots \,|\, S_t = s, A_t = a]$$

$$\tilde{q}_*(s,a) = \sum_{s',r} p(s',r\,|\,s,a)\left[r - \bar{r} + \max_{a'} \tilde{q}_*(s',a')\right]$$

Discounted Q-learning

$$Q_{t+1}(S_t,A_t) \doteq Q_t(S_t,A_t) + \alpha_t\underbrace{\left[R_{t+1} + \gamma \max_{a'} Q_t(S_{t+1},a') - Q_t(S_t,A_t)\right]}_{\delta^\gamma_t}$$

$$Q_{t+1}(S_t,A_t) \doteq Q_t(S_t,A_t) + \alpha_t\underbrace{\left[R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1},a') - Q_t(S_t,A_t)\right]}_{\delta_t}$$

new_estimate = old_estimate + stepsize * (new_target - old_estimate)

ESTIMATING THE VALUES FROM DATA

$S_0\ A_0\ R_1\ S_1\ A_1, R_2 \dots\ S_t\ A_t\ R_{t+1}\ S_{t+1}\ A_{t+1}\ R_{t+2}\ \dots$

$$q^\gamma_\pi(s,a) \doteq \mathbb{E}_\pi[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots \mid S_t = s, A_t = a]$$

$$q^\gamma_*(s,a) = \sum_{s',r} p(s', r \mid s, a)\big[r + \gamma \max_{a'} q^\gamma_*(s', a')\big]$$

$$\tilde{q}_\pi(s,a) \doteq \mathbb{E}_\pi[R_{t+1} - r(\pi) + R_{t+2} - r(\pi) + \dots \mid S_t = s, A_t = a]$$

$$\tilde{q}_*(s,a) = \sum_{s',r} p(s', r \mid s, a)\big[r - \bar{r} + \max_{a'} \tilde{q}_*(s', a')\big]$$

Discounted Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \big[\underbrace{R_{t+1} + \gamma \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)}_{\delta^\gamma_t}\big]$$

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \big[\underbrace{R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)}_{\delta_t}\big]$$

new_estimate = old_estimate + stepsize * (new_target - old_estimate)

ESTIMATING THE VALUES FROM DATA

$$S_0\ A_0\ R_1\ S_1\ A_1, R_2 \ldots\ S_t\ A_t\ R_{t+1}\ S_{t+1}\ A_{t+1}\ R_{t+2}\ \ldots$$

$$q^{\gamma}_{\pi}(s,a) \doteq \mathbb{E}_{\pi}[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots \mid S_t = s, A_t = a]$$

$$q^{\gamma}_{*}(s,a) = \sum_{s',r} p(s',r \mid s,a)\left[r + \gamma \max_{a'} q^{\gamma}_{*}(s',a')\right]$$

$$\tilde{q}_{\pi}(s,a) \doteq \mathbb{E}_{\pi}[R_{t+1} - r(\pi) + R_{t+2} - r(\pi) + \ldots \mid S_t = s, A_t = a]$$

$$\tilde{q}_{*}(s,a) = \sum_{s',r} p(s',r \mid s,a)\left[r - \bar{r} + \max_{a'} \tilde{q}_{*}(s',a')\right]$$

Discounted Q-learning

$$Q_{t+1}(S_t,A_t) \doteq Q_t(S_t,A_t) + \alpha_t \underbrace{\left[R_{t+1} + \gamma \max_{a'} Q_t(S_{t+1},a') - Q_t(S_t,A_t)\right]}_{\delta^{\gamma}_t}$$

$$Q_{t+1}(S_t,A_t) \doteq Q_t(S_t,A_t) + \alpha_t \underbrace{\left[R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1},a') - Q_t(S_t,A_t)\right]}_{\delta_t}$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \beta_t \delta_t$$

new_estimate = old_estimate + stepsize * (new_target - old_estimate)

ESTIMATING THE VALUES FROM DATA

$$S_0\ A_0\ R_1\ S_1\ A_1, R_2 \ldots\ S_t\ A_t\ R_{t+1}\ S_{t+1}\ A_{t+1}\ R_{t+2}\ \ldots$$

$$q_\pi^\gamma(s,a) \doteq \mathbb{E}_\pi[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots \mid S_t = s, A_t = a]$$

$$q_*^\gamma(s,a) = \sum_{s',r} p(s',r \mid s,a)\left[r + \gamma \max_{a'} q_*^\gamma(s',a')\right]$$

$$\tilde{q}_\pi(s,a) \doteq \mathbb{E}_\pi[R_{t+1} - r(\pi) + R_{t+2} - r(\pi) + \ldots \mid S_t = s, A_t = a]$$

$$\tilde{q}_*(s,a) = \sum_{s',r} p(s',r \mid s,a)\left[r - \bar{r} + \max_{a'} \tilde{q}_*(s',a')\right]$$

Discounted Q-learning

$$Q_{t+1}(S_t,A_t) \doteq Q_t(S_t,A_t) + \alpha_t \underbrace{\left[R_{t+1} + \gamma \max_{a'} Q_t(S_{t+1},a') - Q_t(S_t,A_t)\right]}_{\delta_t^\gamma}$$

Differential Q-learning

$$Q_{t+1}(S_t,A_t) \doteq Q_t(S_t,A_t) + \alpha_t \underbrace{\left[R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1},a') - Q_t(S_t,A_t)\right]}_{\delta_t}$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \beta_t \delta_t$$

new_estimate = old_estimate + stepsize * (new_target - old_estimate)

ESTIMATING THE AVERAGE REWARD FROM DATA

$S_0 \; A_0 \; R_1 \; S_1 \; A_1, R_2 \ldots \; S_t \; A_t \; R_{t+1} \; S_{t+1} \; A_{t+1} \; R_{t+2} \; \ldots$

Differential Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \underbrace{\Big[R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)\Big]}_{\delta_t}$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \beta_t \delta_t$$

new_estimate = old_estimate + stepsize * (new_target - old_estimate)

ESTIMATING THE AVERAGE REWARD FROM DATA

$S_0 \; A_0 \; R_1 \; S_1 \; A_1, R_2 \dots \; S_t \; A_t \; R_{t+1} \; S_{t+1} \; A_{t+1} \; R_{t+2} \; \dots$

Differential Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \underbrace{\Big[R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)\Big]}_{\delta_t}$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \beta_t \delta_t$$

$$\tilde{q}_*(s, a) = \sum_{s', r} p(s', r \mid s, a)\Big[r - \bar{r} + \max_{a'} \tilde{q}_*(s', a')\Big]$$

new_estimate = old_estimate + stepsize * (new_target - old_estimate)

ESTIMATING THE AVERAGE REWARD FROM DATA

$$S_0 \; A_0 \; R_1 \; S_1 \; A_1, R_2 \ldots \; S_t \; A_t \; R_{t+1} \; S_{t+1} \; A_{t+1} \; R_{t+2} \; \ldots$$

Differential Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \underbrace{\left[R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)\right]}_{\delta_t}$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \beta_t \delta_t$$

$$\tilde{q}_*(s,a) = \sum_{s',r} p(s', r \mid s, a)\left[r + \max_{a'} \tilde{q}_*(s', a')\right] - \bar{r}$$

new_estimate = old_estimate + stepsize * (new_target - old_estimate)

ESTIMATING THE AVERAGE REWARD FROM DATA

$S_0 \; A_0 \; R_1 \; S_1 \; A_1, R_2 \dots \; S_t \; A_t \; R_{t+1} \; S_{t+1} \; A_{t+1} \; R_{t+2} \; \dots$

Differential Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \underbrace{\left[R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)\right]}_{\delta_t}$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \beta_t \delta_t$$

$$\bar{r} = \sum_{s',r} p(s', r \mid s, a)\left[r + \max_{a'} \tilde{q}_*(s', a')\right] - \tilde{q}_*(s, a)$$

new_estimate = old_estimate + stepsize * (new_target - old_estimate)

ESTIMATING THE AVERAGE REWARD FROM DATA

$S_0\ A_0\ R_1\ S_1\ A_1, R_2 \dots\ S_t\ A_t\ R_{t+1}\ S_{t+1}\ A_{t+1}\ R_{t+2}\ \dots$

Differential Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \underbrace{\left[R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)\right]}_{\delta_t}$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \beta_t \delta_t$$

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ESTIMATING THE AVERAGE REWARD FROM DATA

$S_0\ A_0\ R_1\ S_1\ A_1, R_2 \ldots\ S_t\ A_t\ R_{t+1}\ S_{t+1}\ A_{t+1}\ R_{t+2}\ \ldots$

Differential Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \underbrace{\left[R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)\right]}_{\delta_t}$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \beta_t \delta_t$$

$$\bar{r} = \sum_{s',r} p(s', r \mid s, a)\left[r + \max_{a'} \tilde{q}_*(s', a') - \tilde{q}_*(s, a)\right]$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \beta_t (R_{t+1} + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t) - \bar{R}_t)$$

new_estimate = old_estimate + stepsize * (new_target - old_estimate)

ESTIMATING THE AVERAGE REWARD FROM DATA

$S_0\ A_0\ R_1\ S_1\ A_1, R_2 \dots\ S_t\ A_t\ R_{t+1}\ S_{t+1}\ A_{t+1}\ R_{t+2}\ \dots$

Differential Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \underbrace{\left[R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)\right]}_{\delta_t}$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \beta_t \delta_t$$

$$\bar{r} = \sum_{s', r} p(s', r \mid s, a)\left[r + \max_{a'} \tilde{q}_*(s', a') - \tilde{q}_*(s, a)\right]$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \beta_t (R_{t+1} + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t) - \bar{R}_t)$$

new_estimate = old_estimate + stepsize * (new_target - old_estimate)

ESTIMATING THE AVERAGE REWARD FROM DATA

$S_0\ A_0\ R_1\ S_1\ A_1, R_2 \dots\ S_t\ A_t\ R_{t+1}\ S_{t+1}\ A_{t+1}\ R_{t+2}\ \dots$

Differential Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \big[\underbrace{R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)}_{\delta_t} \big]$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \beta_t \delta_t$$

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$$\bar{R}_{t+1} \doteq \bar{R}_t + \beta_t \big(R_{t+1} + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t) - \bar{R}_t \big)$$

new_estimate = old_estimate + stepsize * (new_target - old_estimate)

ESTIMATING THE AVERAGE REWARD FROM DATA

$S_0\ A_0\ R_1\ S_1\ A_1, R_2 \dots\ S_t\ A_t\ R_{t+1}\ S_{t+1}\ A_{t+1}\ R_{t+2}\ \dots$

Differential Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \underbrace{\left[R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)\right]}_{\delta_t}$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \beta_t \delta_t$$

$$\bar{r} = \sum_{s',r} p(s', r \mid s, a)\left[r + \max_{a'} \tilde{q}_*(s', a') - \tilde{q}_*(s, a)\right]$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \beta_t \underbrace{(R_{t+1} + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t) - \bar{R}_t)}_{\delta_t}$$

new_estimate = old_estimate + stepsize * (new_target - old_estimate)

(CONVERGENT) ALGORITHMS FOR OFF-POLICY CONTROL

Differential Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \underbrace{\big[R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)\big]}_{\delta_t}$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \eta \alpha_t \, \delta_t$$

(CONVERGENT) ALGORITHMS FOR OFF-POLICY CONTROL

Differential Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \underbrace{\left[R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)\right]}_{\delta_t}$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \underbrace{\eta \alpha_t}_{\beta_t} \delta_t$$

(CONVERGENT) ALGORITHMS FOR OFF-POLICY CONTROL

Differential Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \underbrace{\big[R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)\big]}_{\delta_t}$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \underbrace{\eta\alpha_t}_{\beta_t} \delta_t$$

RVI Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \big[R_{t+1} - f(Q_t) + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)\big]$$

(CONVERGENT) ALGORITHMS FOR OFF-POLICY CONTROL

Differential Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \big[\underbrace{R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)}_{\delta_t}\big]$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \underbrace{\eta \alpha_t}_{\beta_t} \delta_t$$

RVI Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \big[R_{t+1} - f(Q_t) + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)\big]$$

(CONVERGENT) ALGORITHMS FOR OFF-POLICY CONTROL

Differential Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \big[\underbrace{R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)}_{\delta_t}\big]$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \underbrace{\eta\alpha_t}_{\beta_t}\delta_t$$

RVI Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \big[R_{t+1} - f(Q_t) + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)\big]$$

Examples of f:

(CONVERGENT) ALGORITHMS FOR OFF-POLICY CONTROL

Differential Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \big[\underbrace{R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)}_{\delta_t} \big]$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \underbrace{\eta \alpha_t}_{\beta_t} \delta_t$$

RVI Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \big[R_{t+1} - f(Q_t) + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t) \big]$$

Examples of f:

- value of a single state–action pair

(CONVERGENT) ALGORITHMS FOR OFF-POLICY CONTROL

Differential Q-learning

$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \big[\underbrace{R_{t+1} - \bar{R}_t + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)}_{\delta_t}\big]$$

$$\bar{R}_{t+1} \doteq \bar{R}_t + \underbrace{\eta \alpha_t}_{\beta_t} \delta_t$$

RVI Q-learning

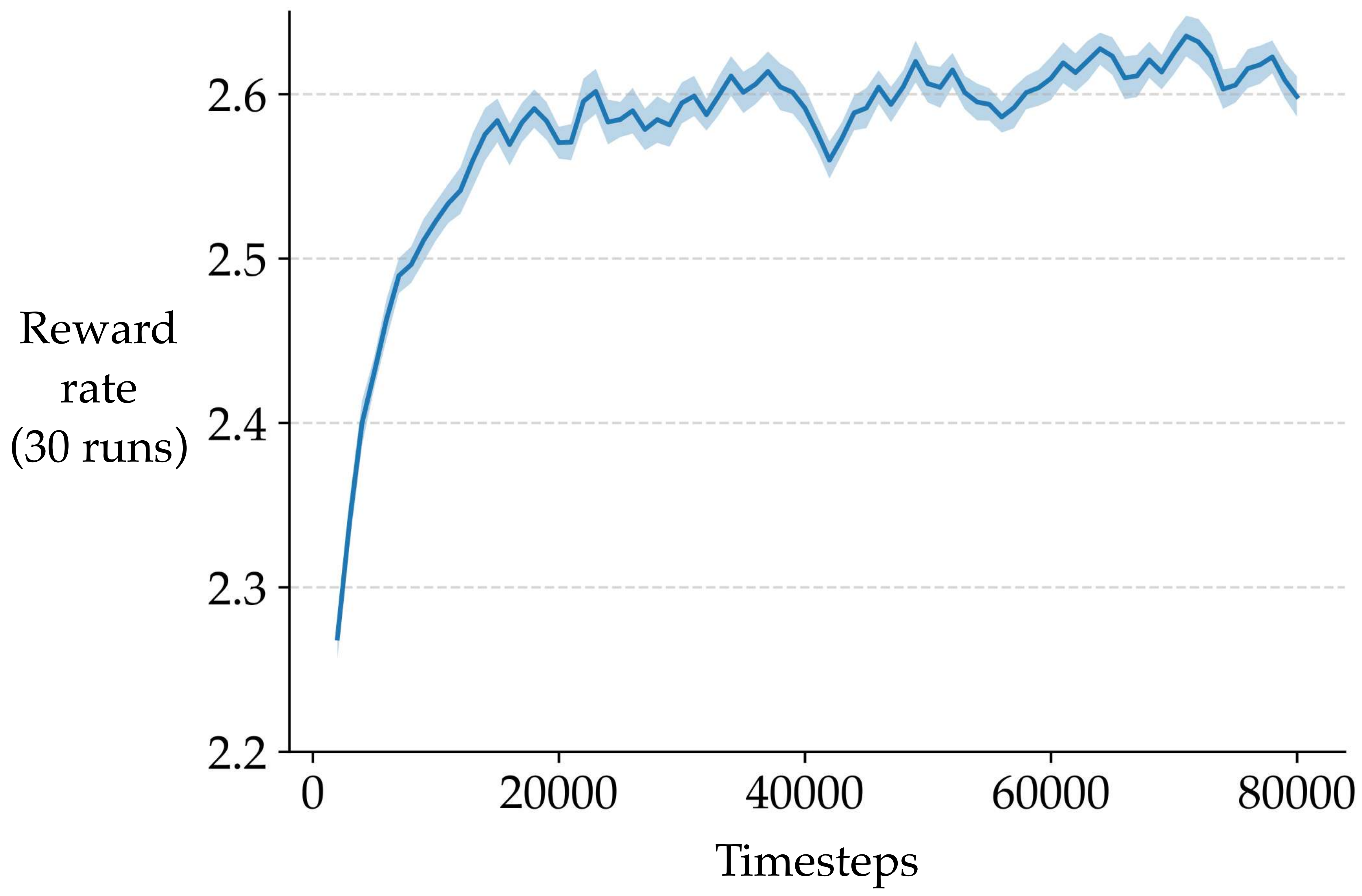
$$Q_{t+1}(S_t, A_t) \doteq Q_t(S_t, A_t) + \alpha_t \big[R_{t+1} - f(Q_t) + \max_{a'} Q_t(S_{t+1}, a') - Q_t(S_t, A_t)\big]$$

Examples of f:

- value of a single state–action pair
- average of values of all state–action pairs

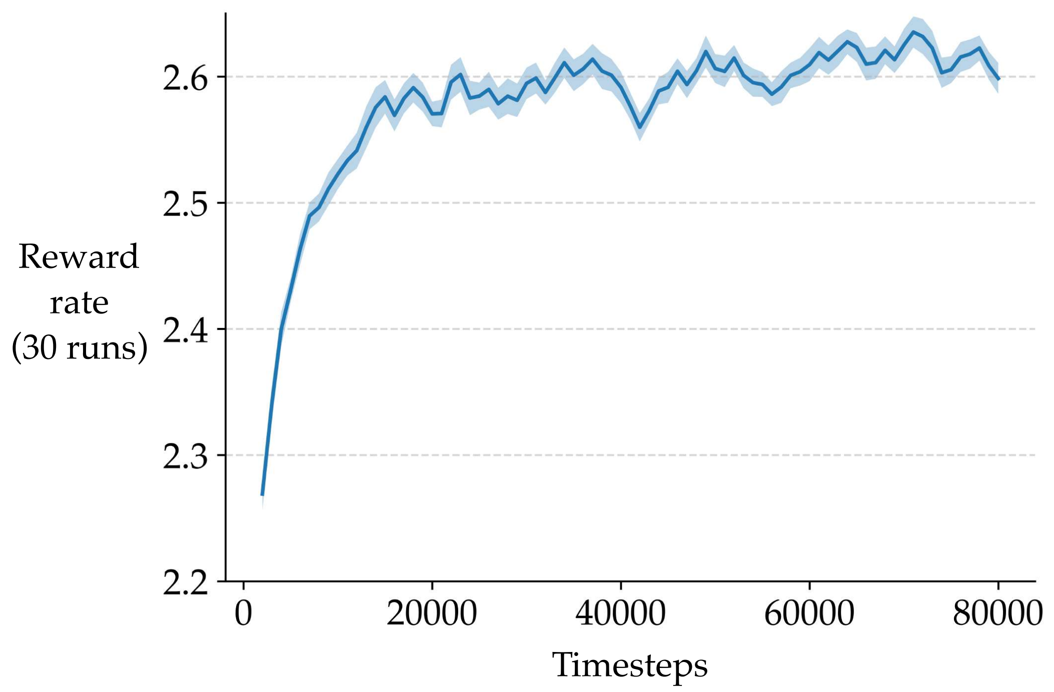
PERFORMANCE COMPARISON

PERFORMANCE COMPARISON



AccessControl

PERFORMANCE COMPARISON



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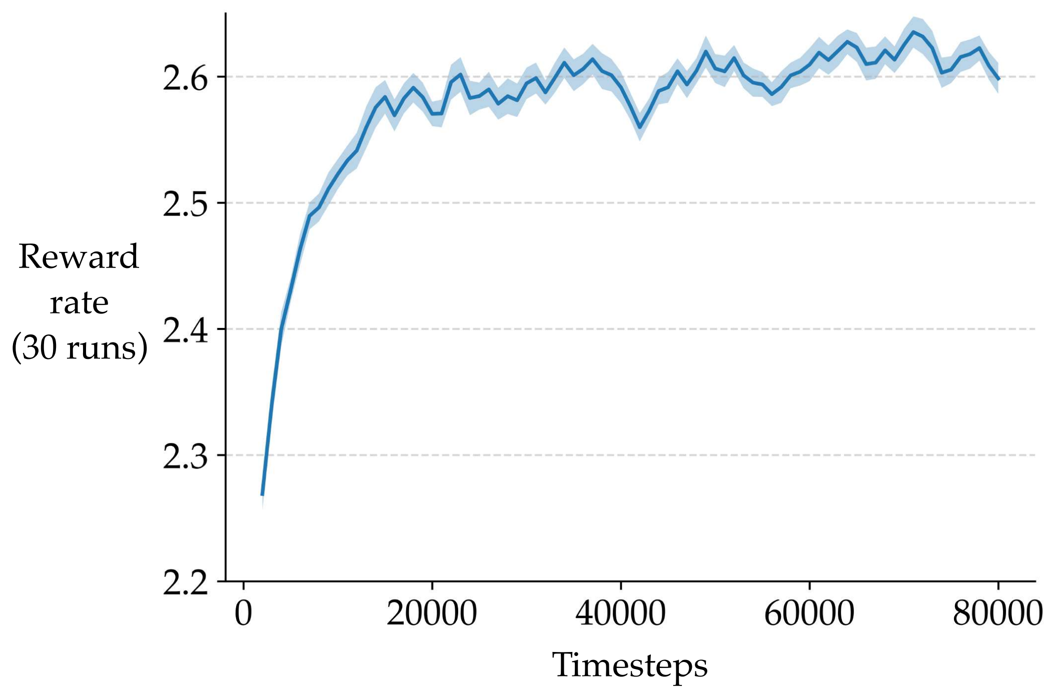
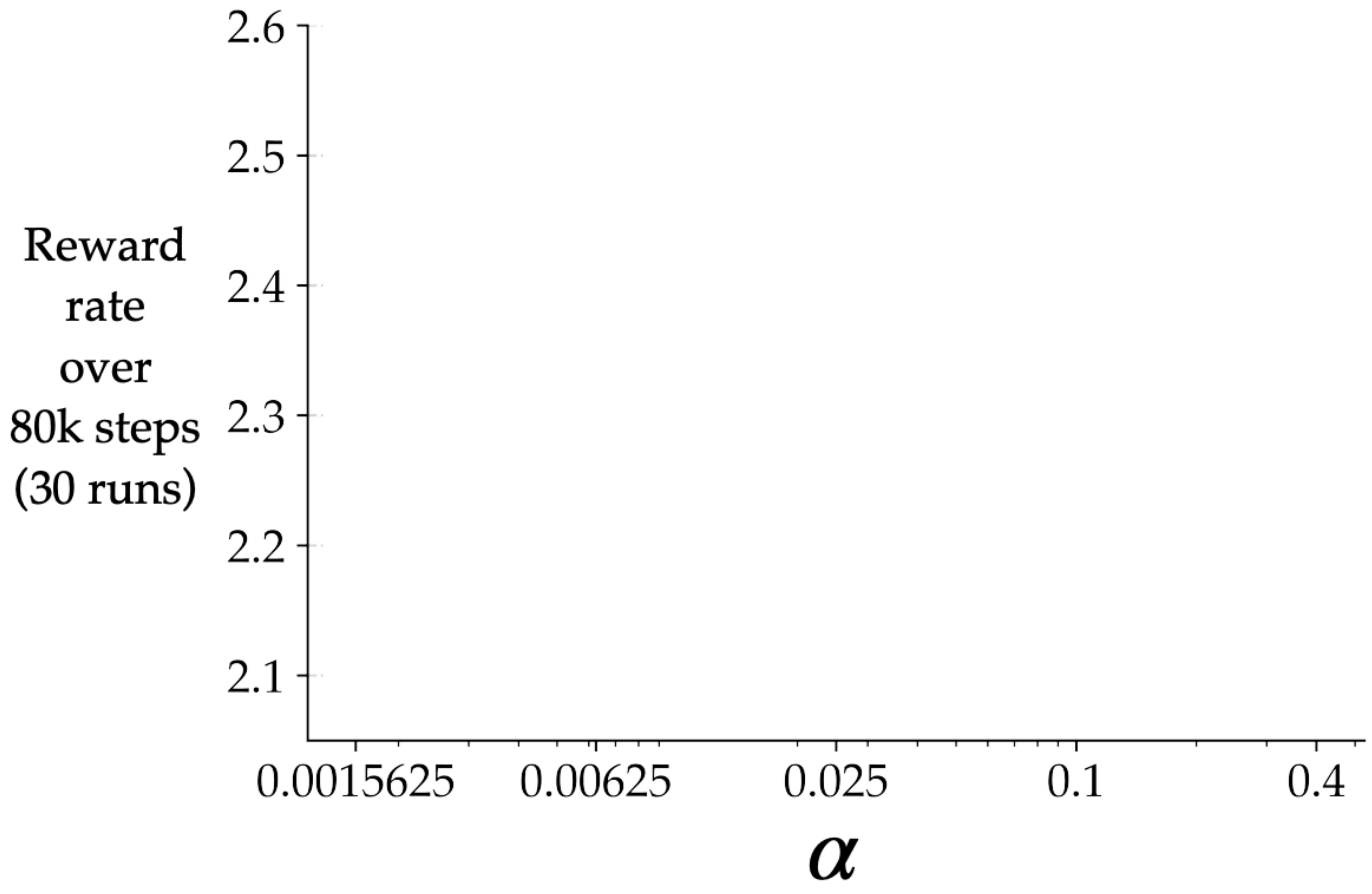
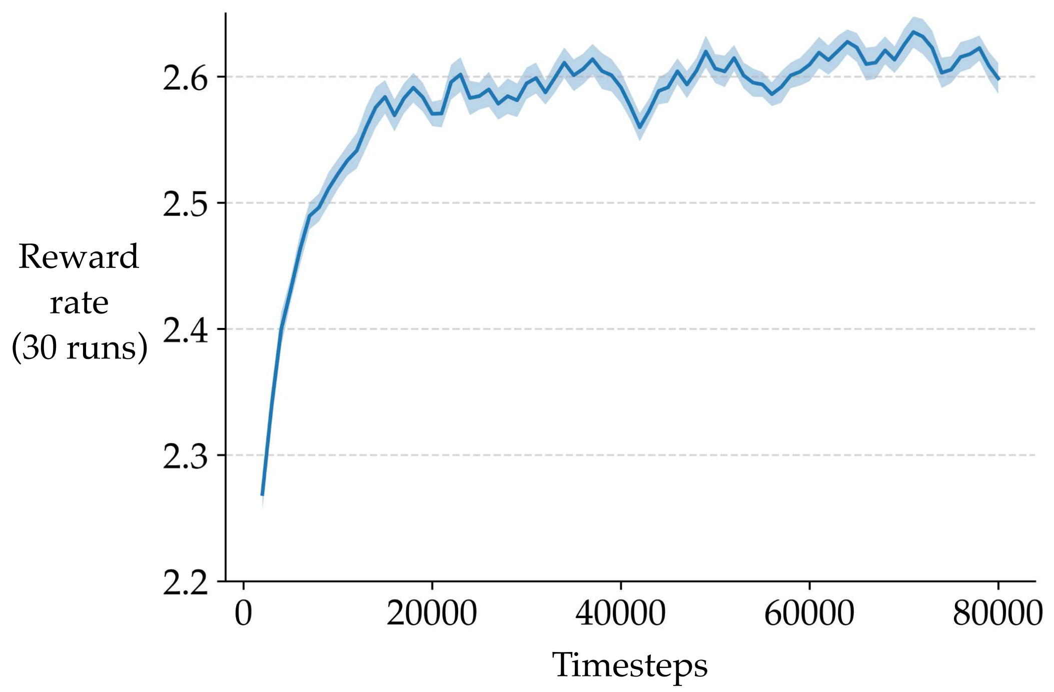
PERFORMANCE COMPARISON

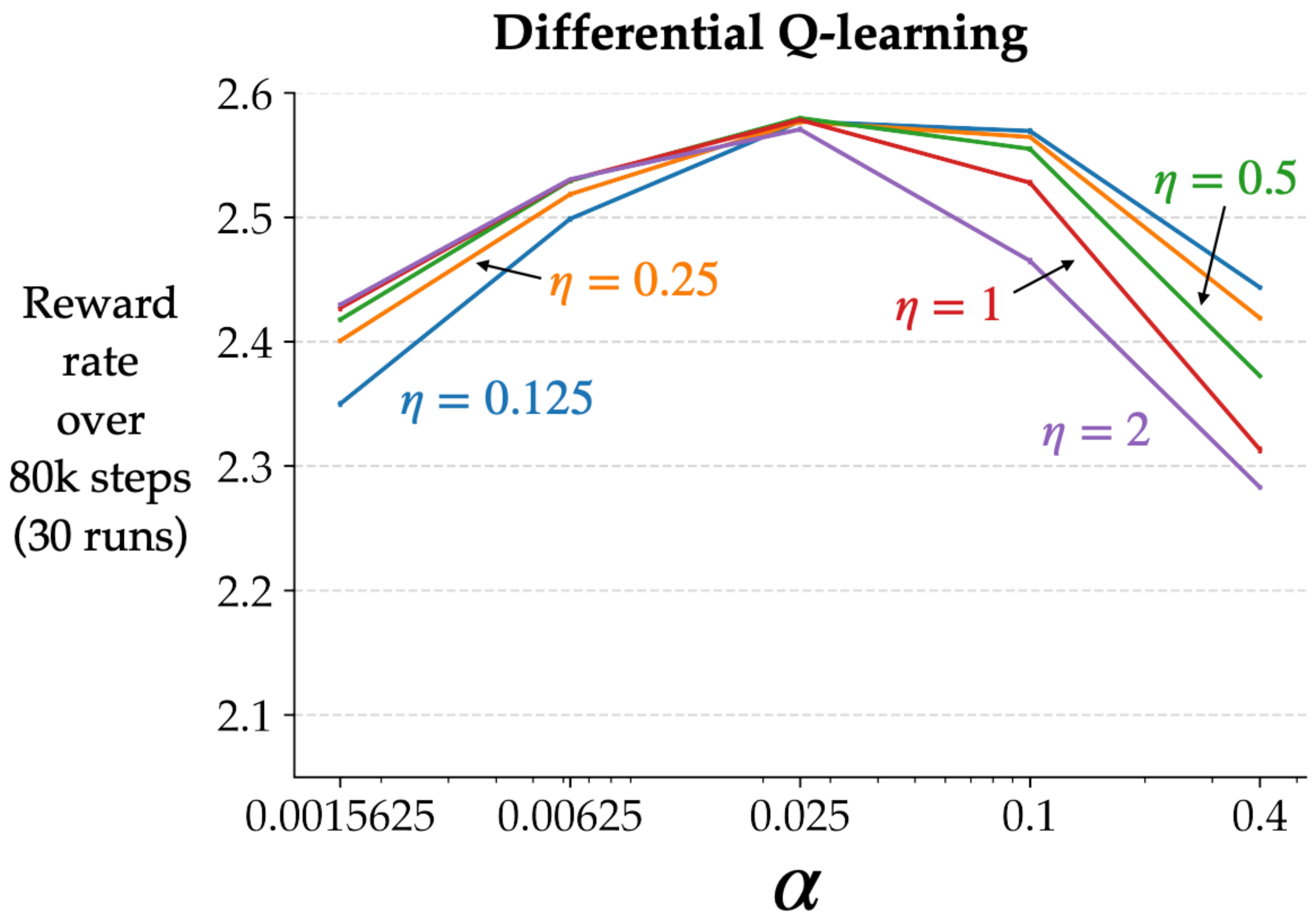




AccessControl

PERFORMANCE COMPARISON





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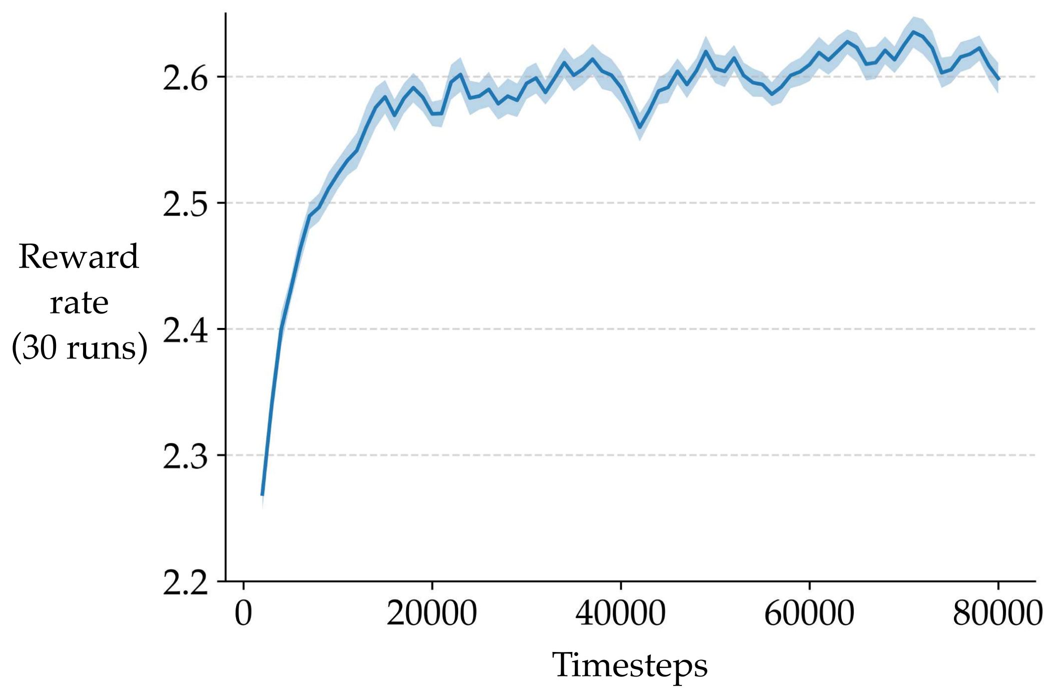

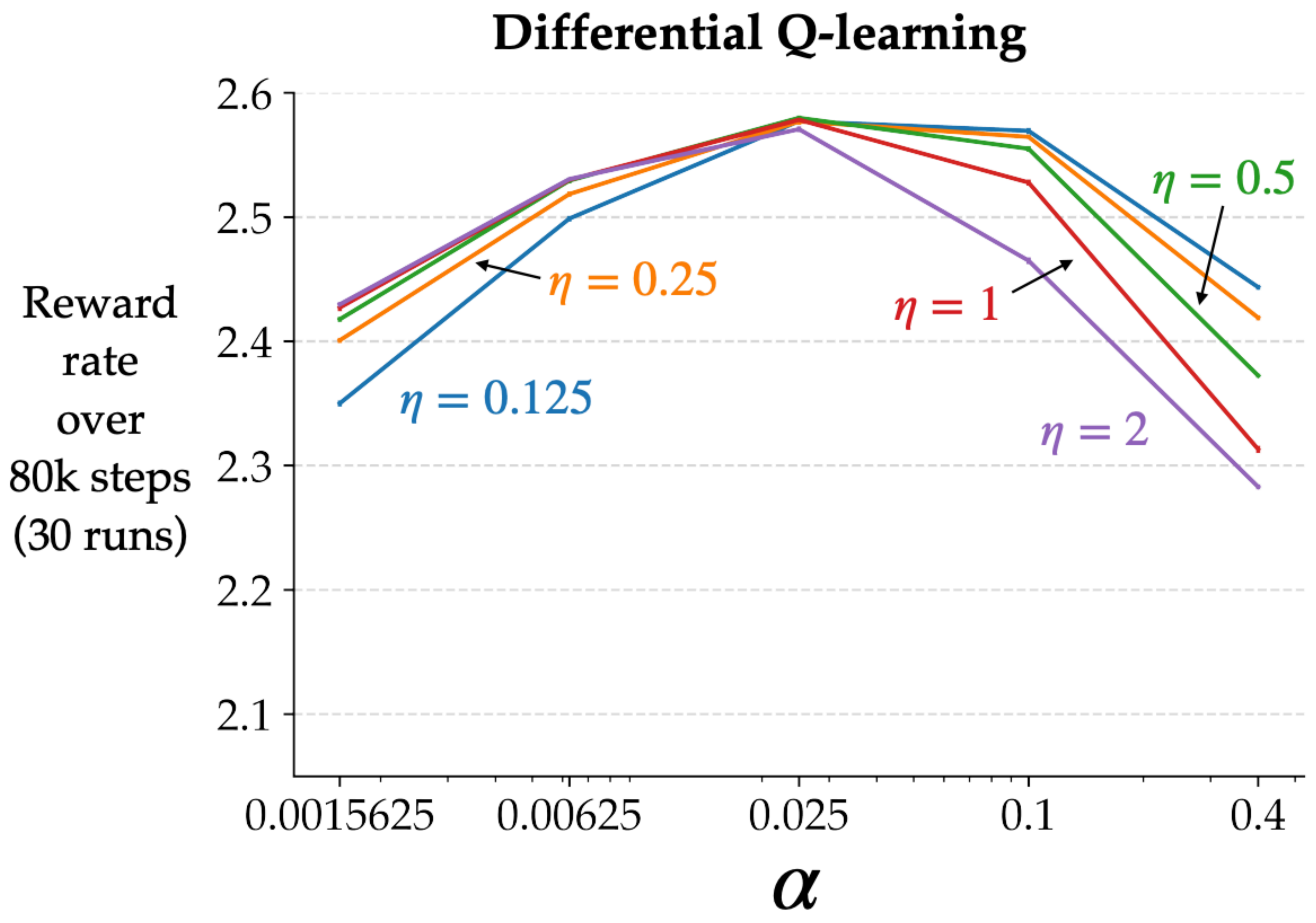

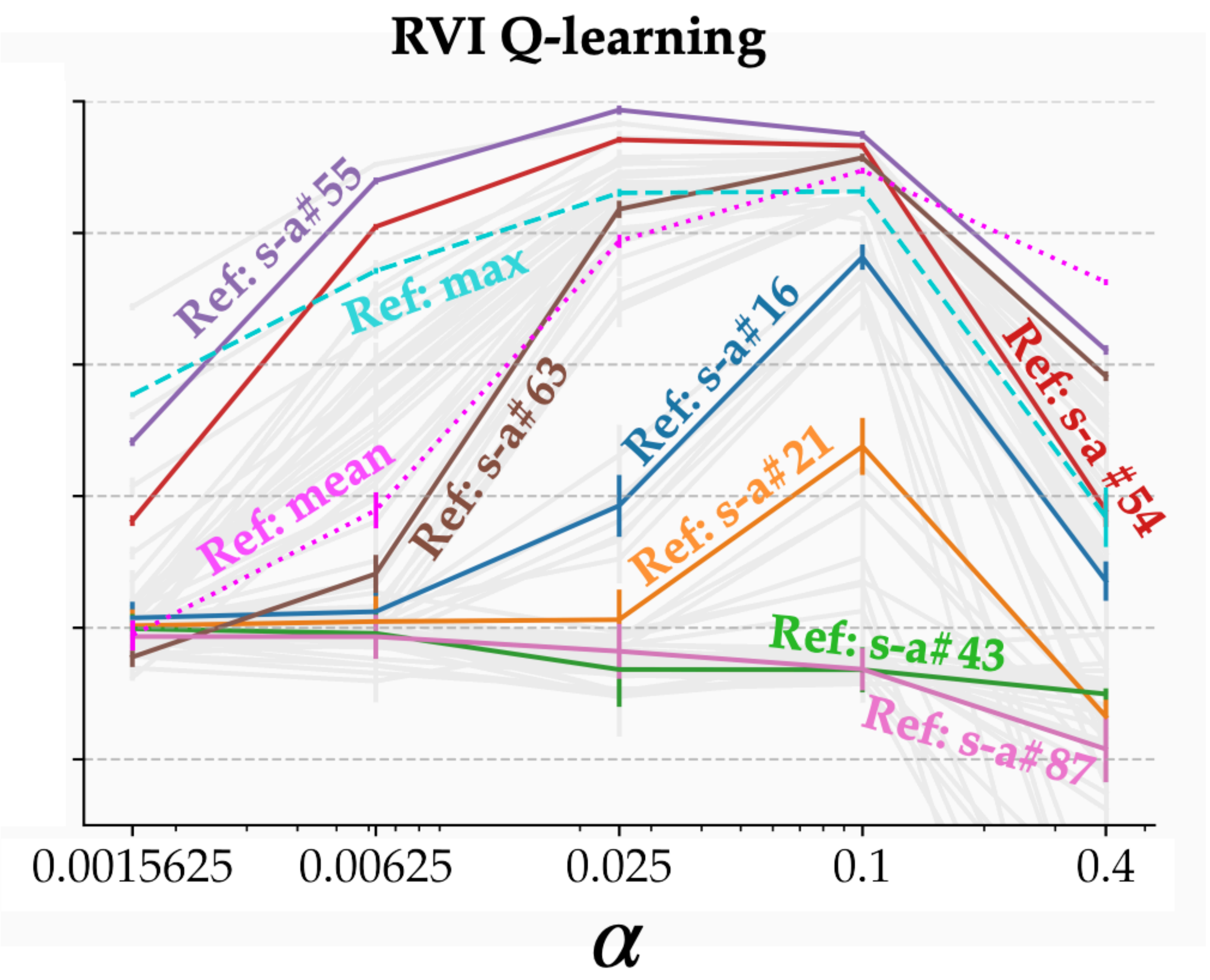

AccessControl

TAKEAWAYS

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OUTLINE

Problem setting

1. *One-step* average-reward methods
2. *Multi-step* average-reward methods
3. An idea to improve *discounted-reward* methods

Conclusions, limitations, and future work

Acknowledgments

OUTLINE

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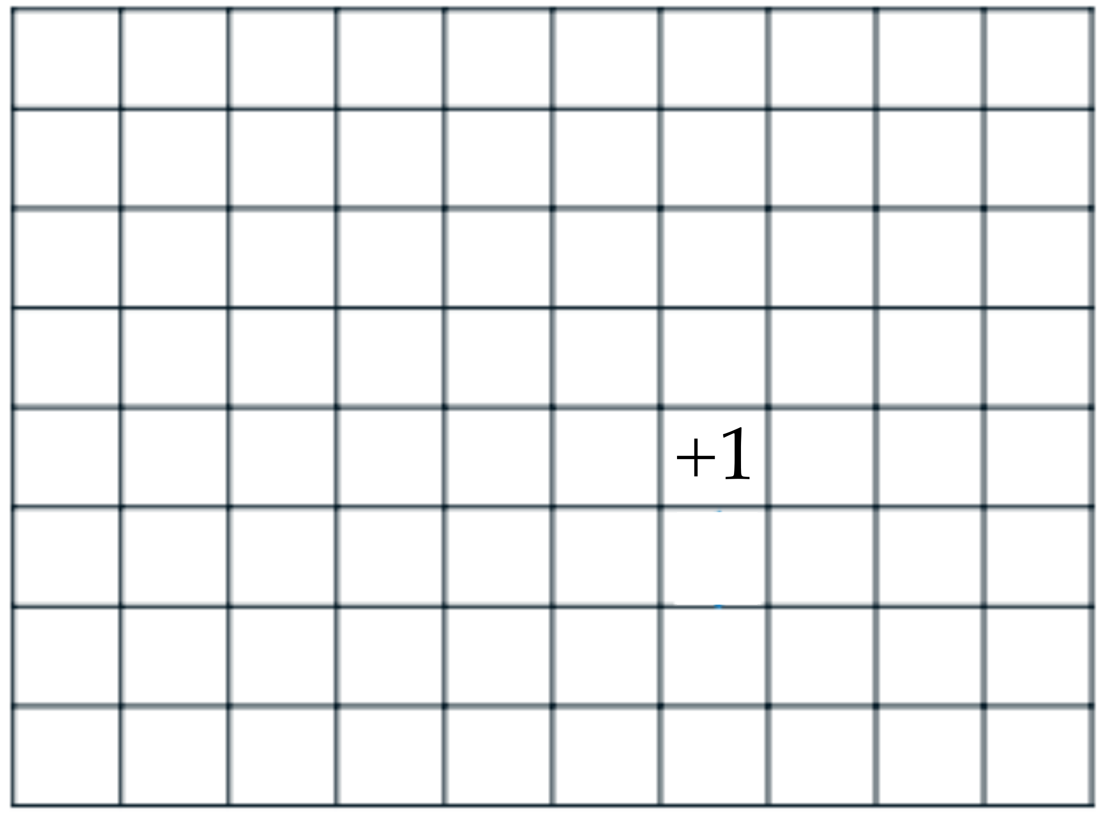

Figures adapted from Sutton and Barto's (2018) Chapter 12

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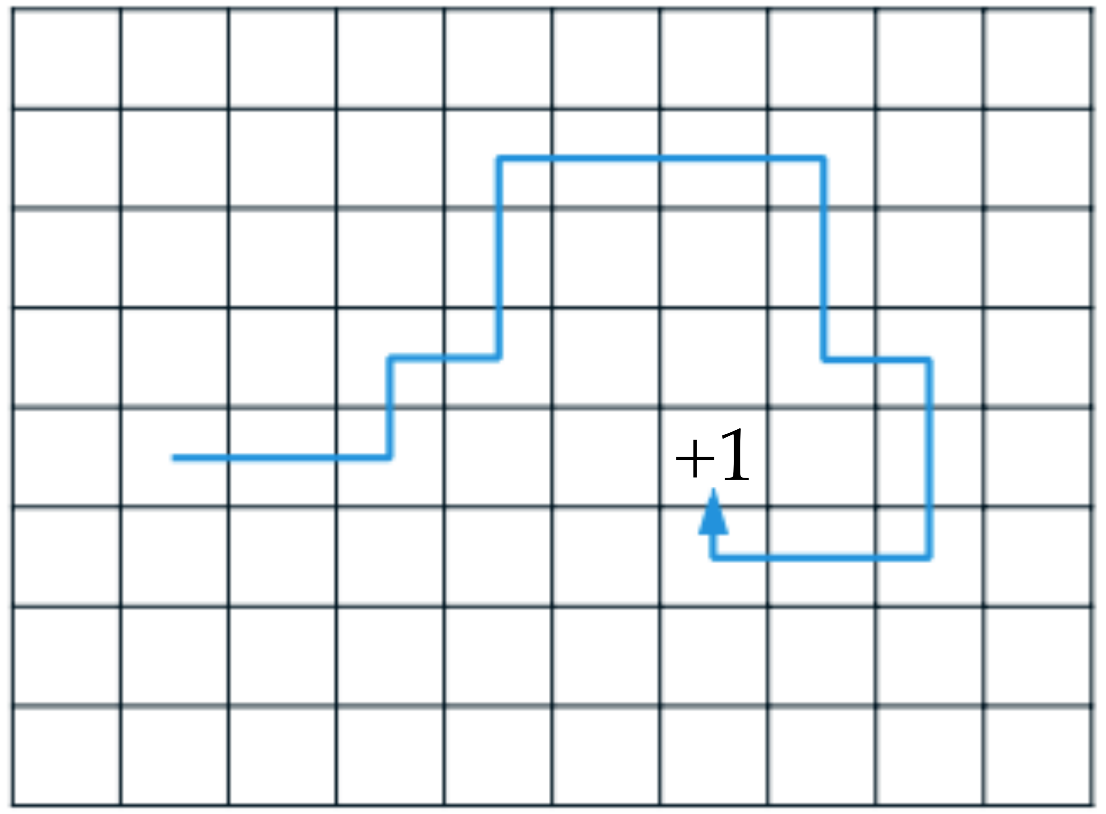

Trajectory

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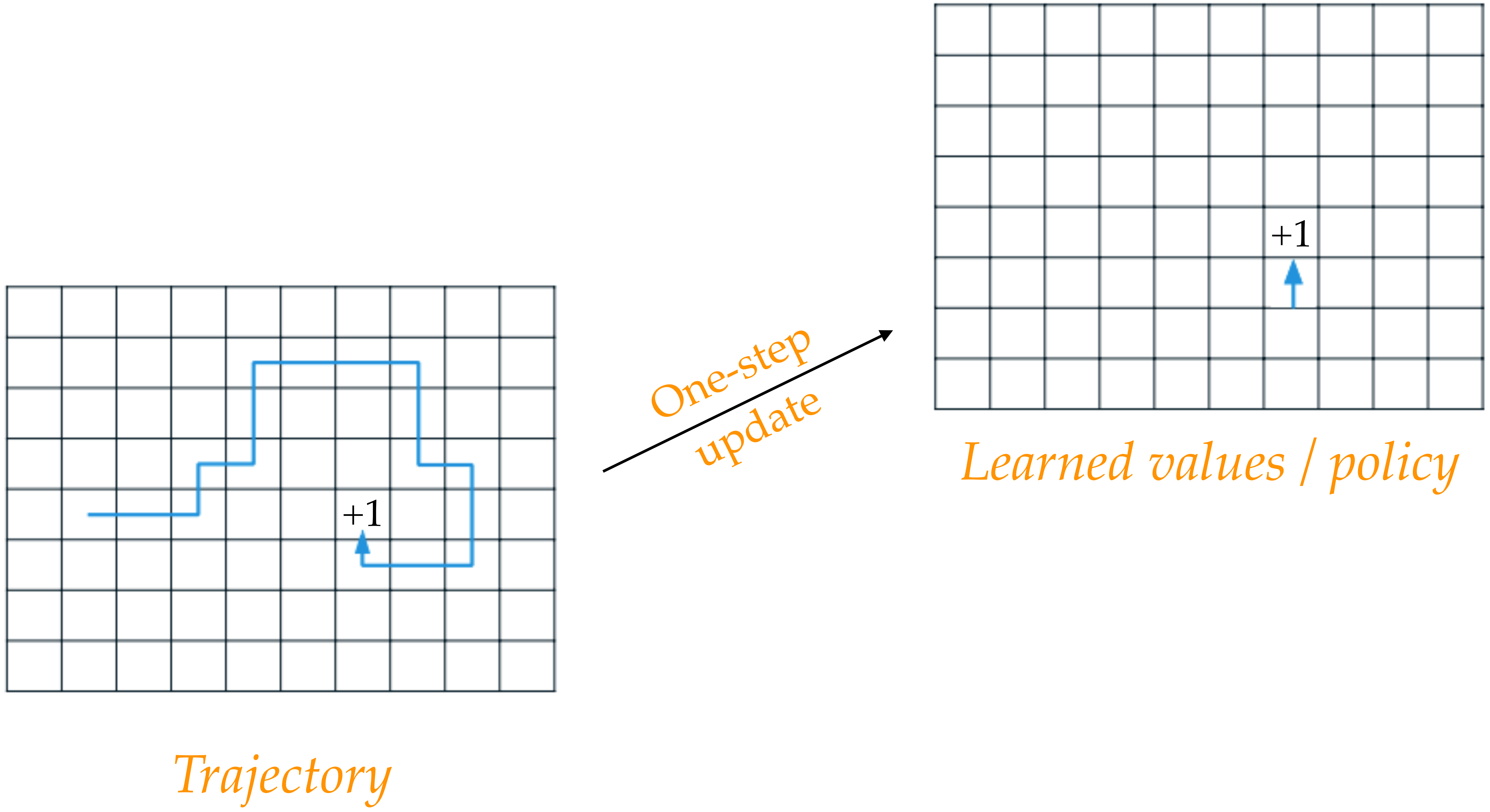

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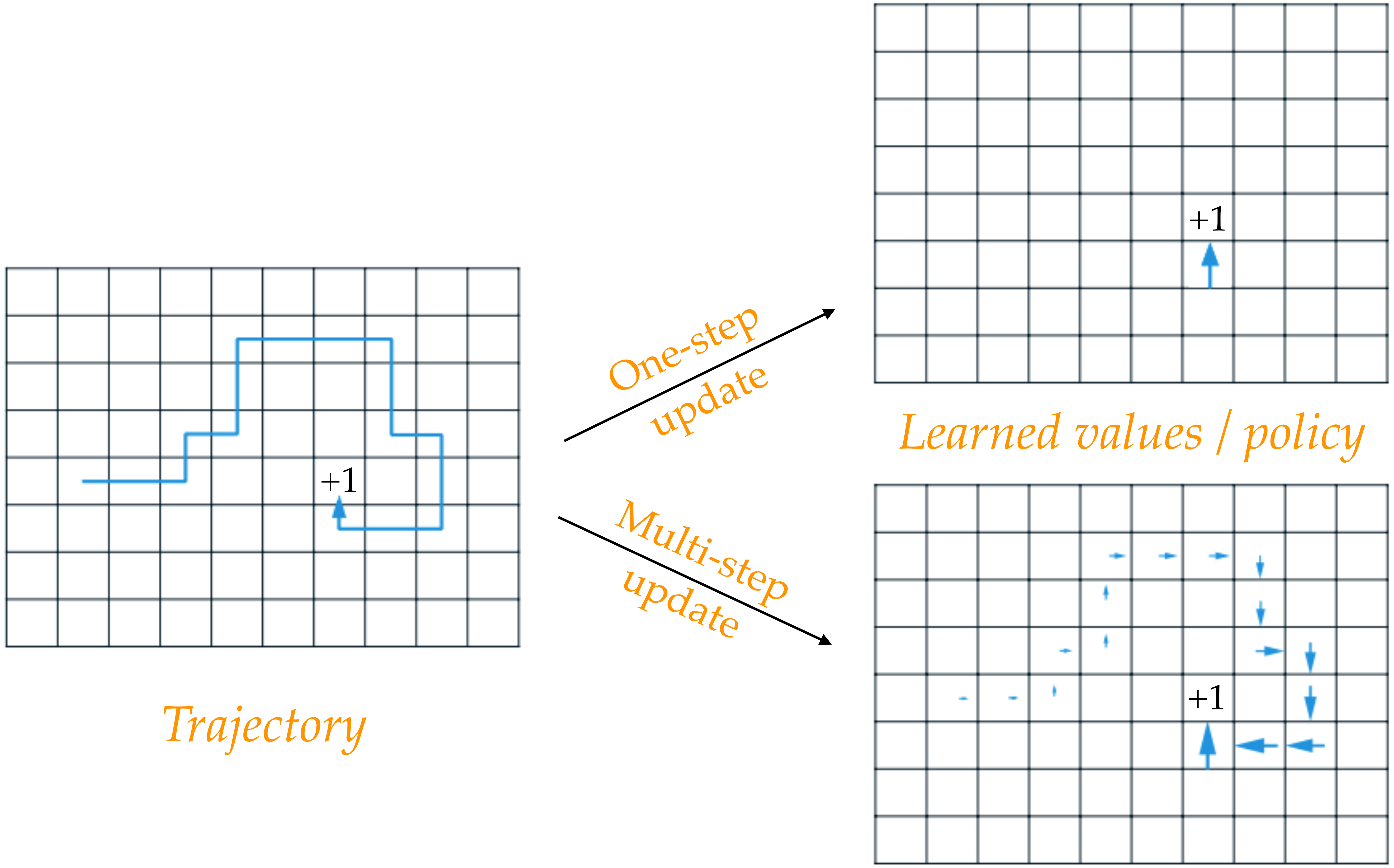

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Guaranteed to converge
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Algorithm 1

Also guaranteed to converge,
under the same conditions

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(WHAT I'VE LEARNED ABOUT)

PROVING CONVERGENCE OF SAMPLED-BASED ALGORITHMS USING THE ODE APPROACH

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1. Show that the sequence of iterates is bounded and asymptotically converges to the solutions of an ODE.

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Proving the convergence of Algorithm 1 was fairly straightforward.

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Algorithm 1*off*

ANALYSIS OF (TABULAR) ALGORITHM 1OFF'S "A" MATRIX

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$$\mathbf{A}^1 \doteq \begin{bmatrix} -\eta & \mathbf{0}^\top \\ \frac{-1}{1-\lambda}\mathbf{D}_\pi \mathbf{1} & \mathbf{D}_\pi(\mathbf{P}_\pi^\lambda - \mathbf{I}) \end{bmatrix}$$

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is Hurwitz.

(Tsitsiklis & Van Roy's (1999) Lemma 7)

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is *not* Hurwitz.

(via a simulation analysis)

ANALYSIS OF (TABULAR) ALGORITHM 1OFF'S "A" MATRIX

$$\mathbf{A}^1 \doteq \begin{bmatrix} -\eta & \mathbf{0}^\top \\ \frac{-1}{1-\lambda}\mathbf{D}_\pi \mathbf{1} & \mathbf{D}_\pi(\mathbf{P}_\pi^\lambda - \mathbb{I}) \end{bmatrix}$$

is Hurwitz.

(Tsitsiklis & Van Roy's (1999) Lemma 7)

$$\mathbf{A}^{1off} \doteq \begin{bmatrix} -\eta & \eta\, \mathbf{d}_b^\top(\mathbf{P}_\pi - \mathbb{I}) \\ \frac{-1}{1-\lambda}\mathbf{D}_b \mathbf{1} & \mathbf{D}_b(\mathbf{P}_\pi^\lambda - \mathbb{I}) \end{bmatrix}$$

is *not* Hurwitz.

(via a simulation analysis)

So Algorithm 1*off* can diverge… :(

EXTENSION TO THE OFF-POLICY SETTING

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One-step *off-policy* Differential TD

$$\mathbf{w}_{t+1} \doteq \mathbf{w}_t + \alpha_t \rho_t \delta_t \mathbf{x}_t$$

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Algorithm 1*off*

ANALYSIS OF (TABULAR) ALGORITHM 2

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OUTLINE

Problem setting

1. *One-step* average-reward methods
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Conclusions, limitations, and future work

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$S_0 \ A_0 \ R_1 \ S_1 \ A_1, R_2 \ldots \ S_t \ A_t \ R_{t+1} \ S_{t+1} \ A_{t+1} \ R_{t+2} \ \ldots$

REWARD CENTERING

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Estimate the average reward and subtract it from the observed rewards

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$$\downarrow$$

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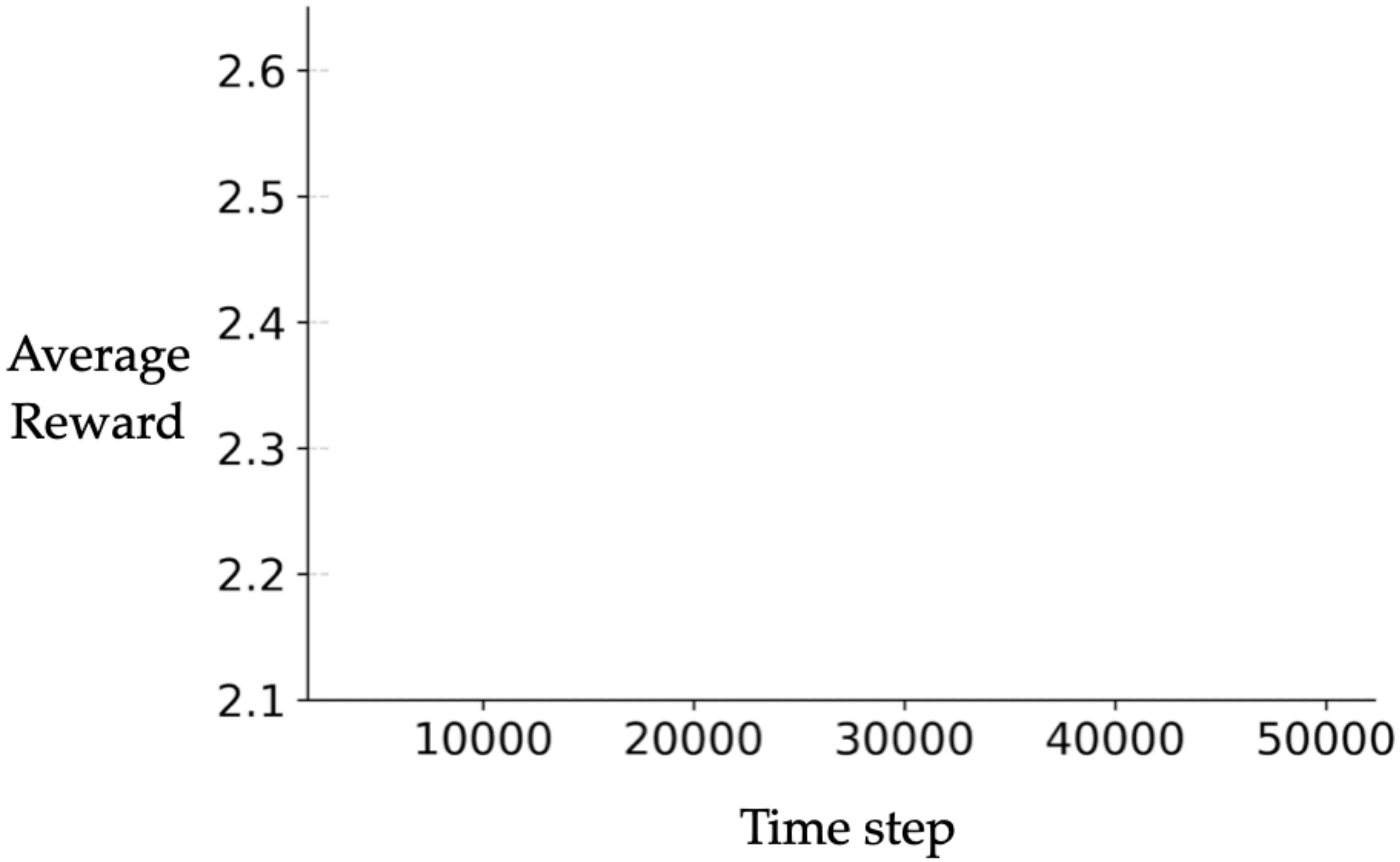
NO INSTABILITY WITH LARGE DISCOUNT FACTORS



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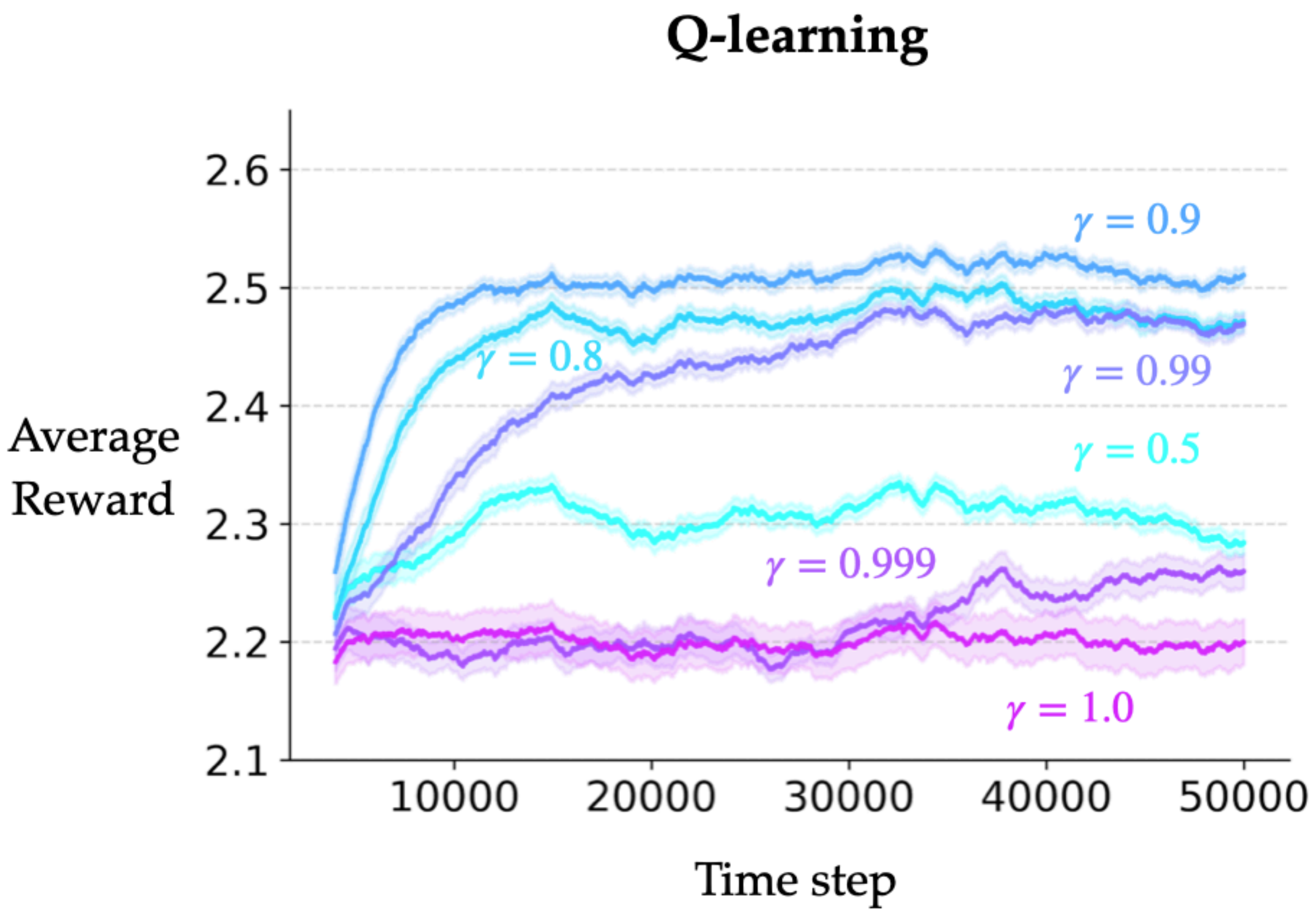

AccessControl (tabular)

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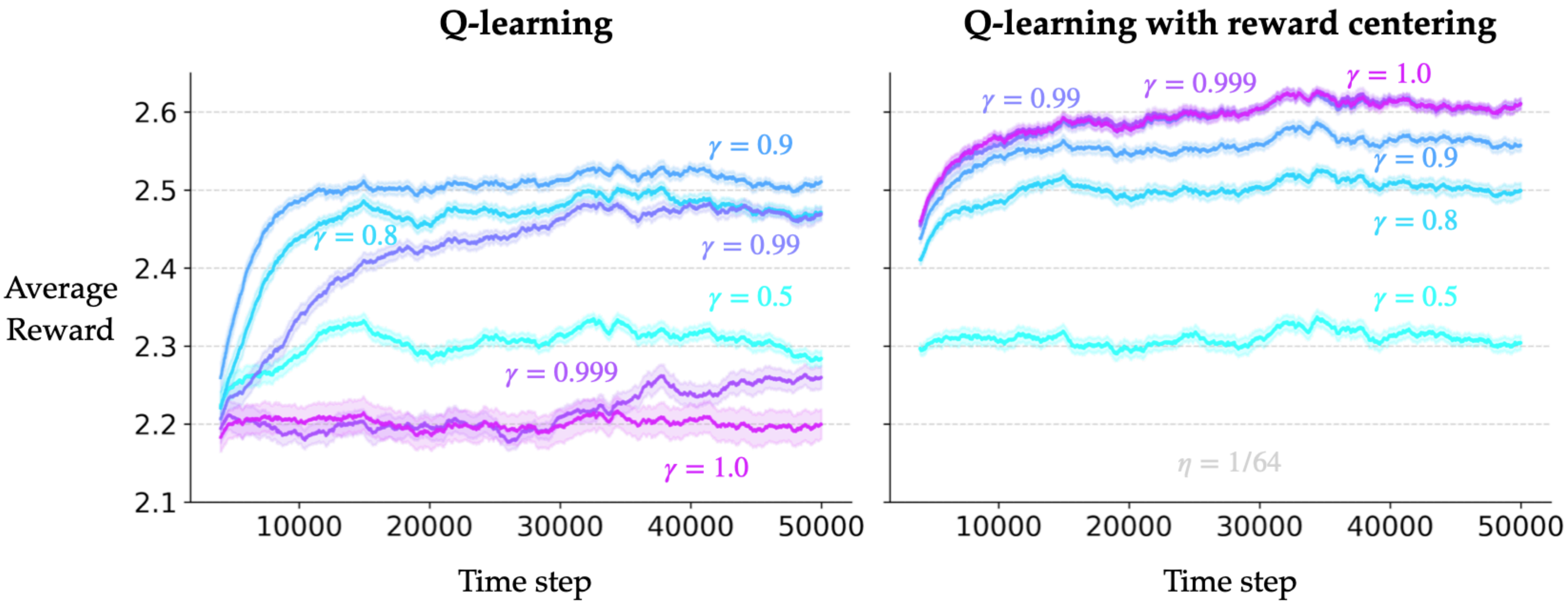

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PuckWorld (linear FA)

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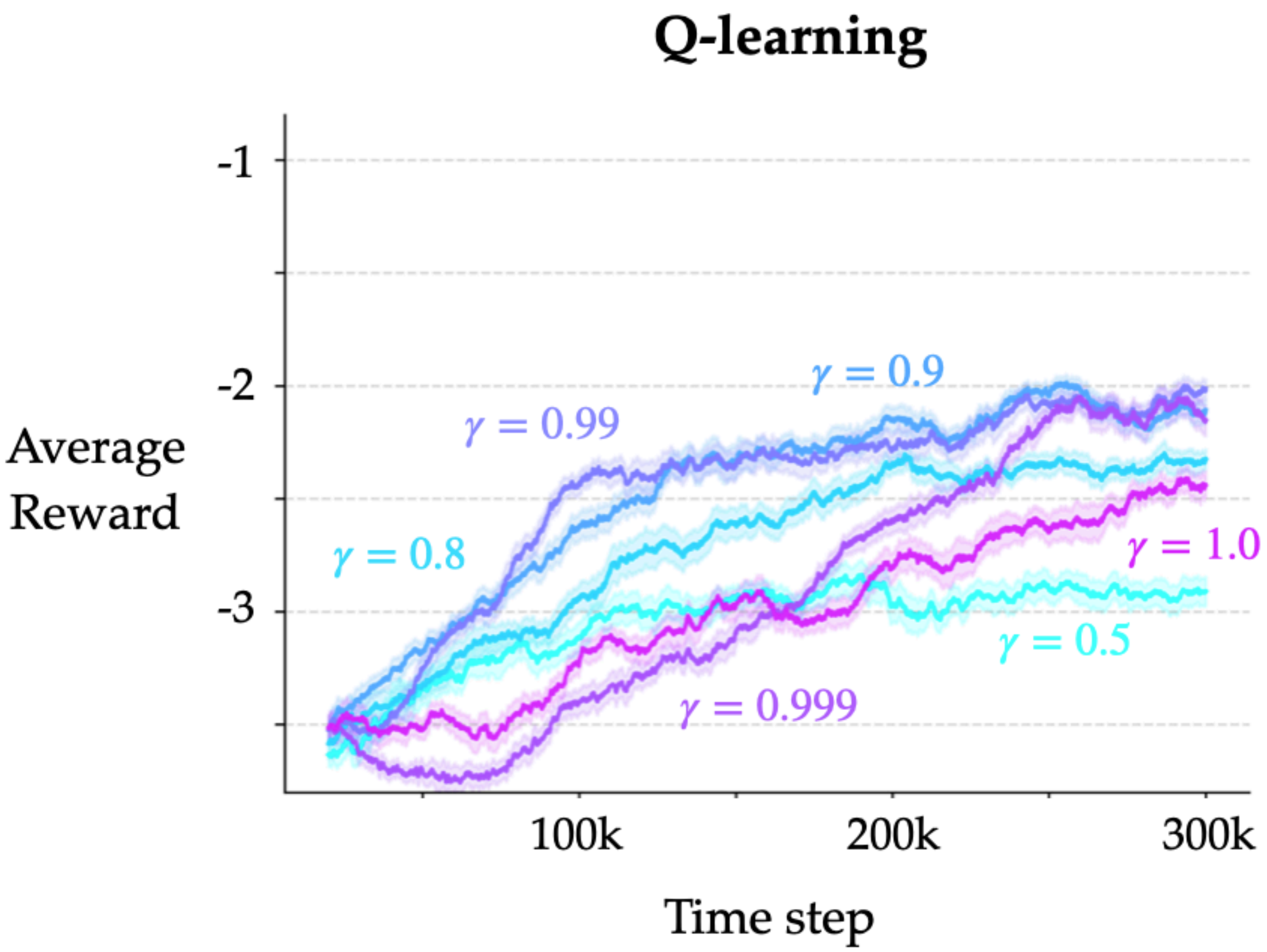

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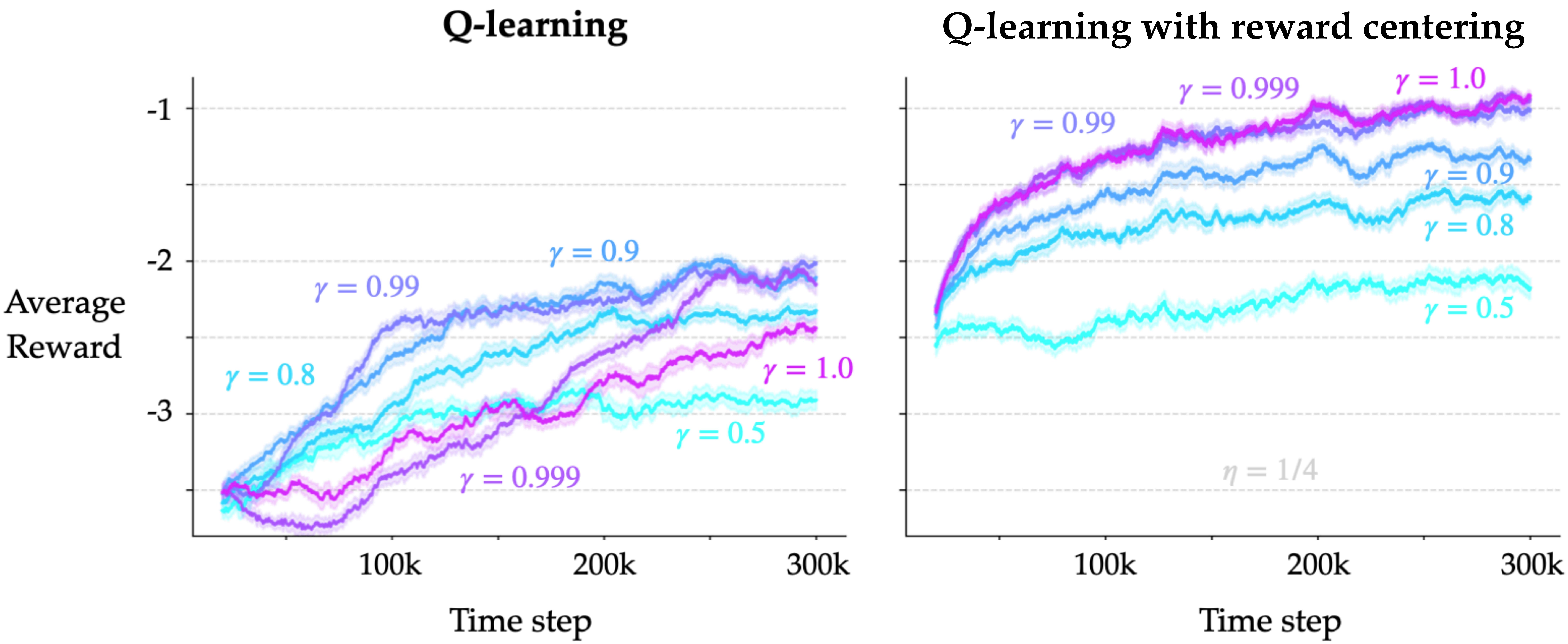

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Pendulum (non-linear FA)

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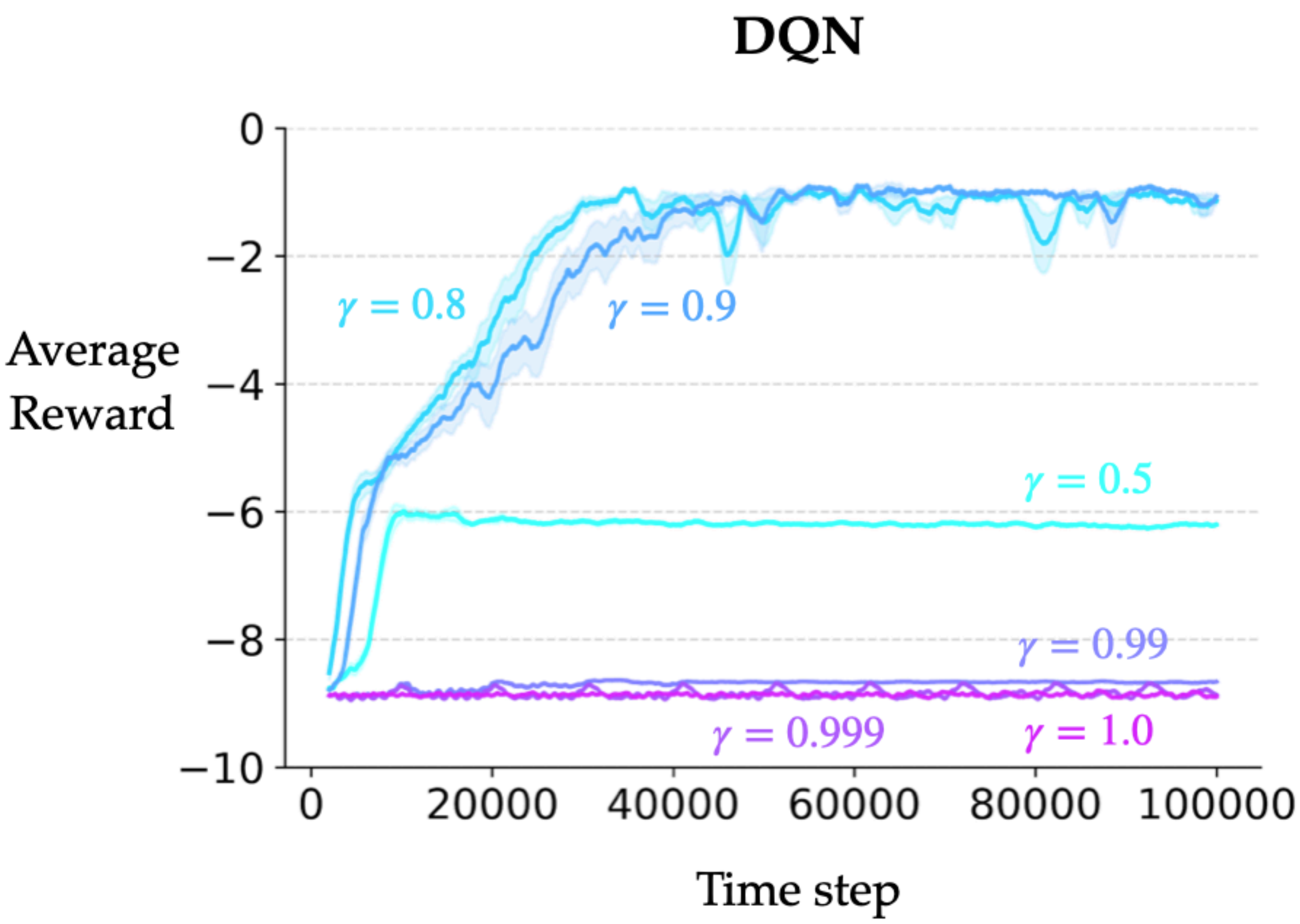

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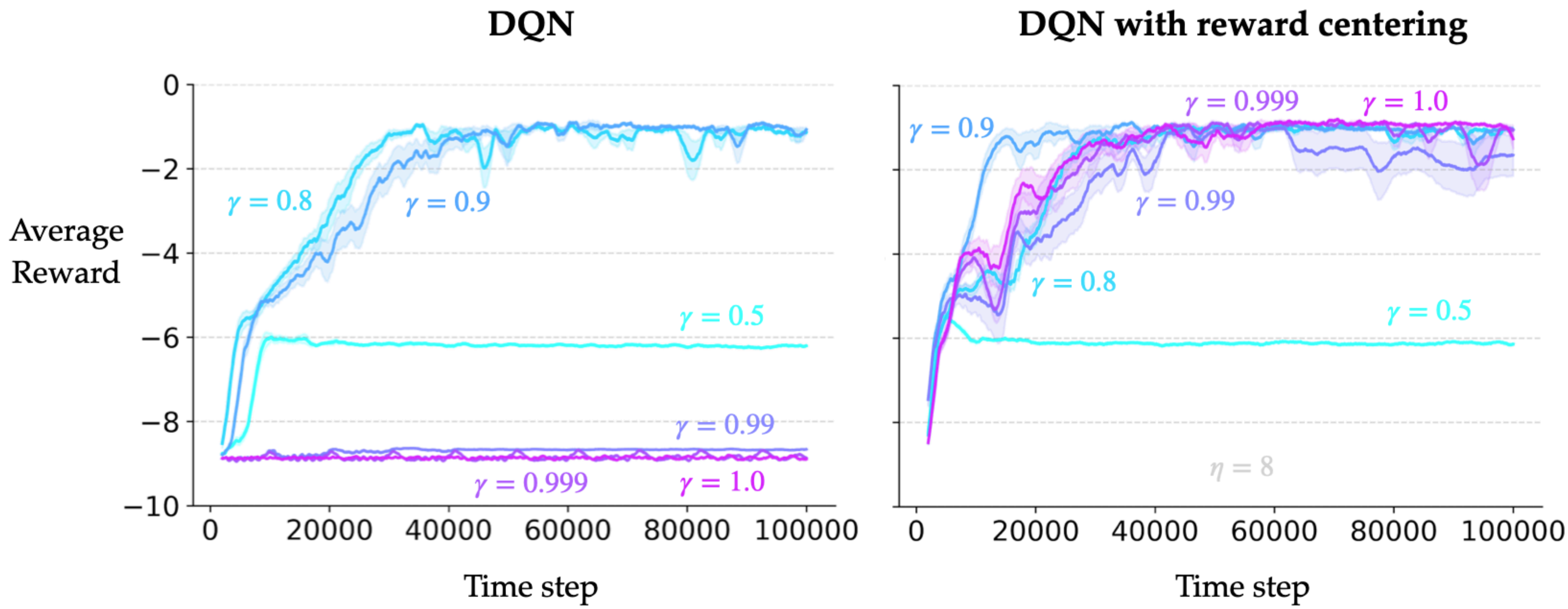

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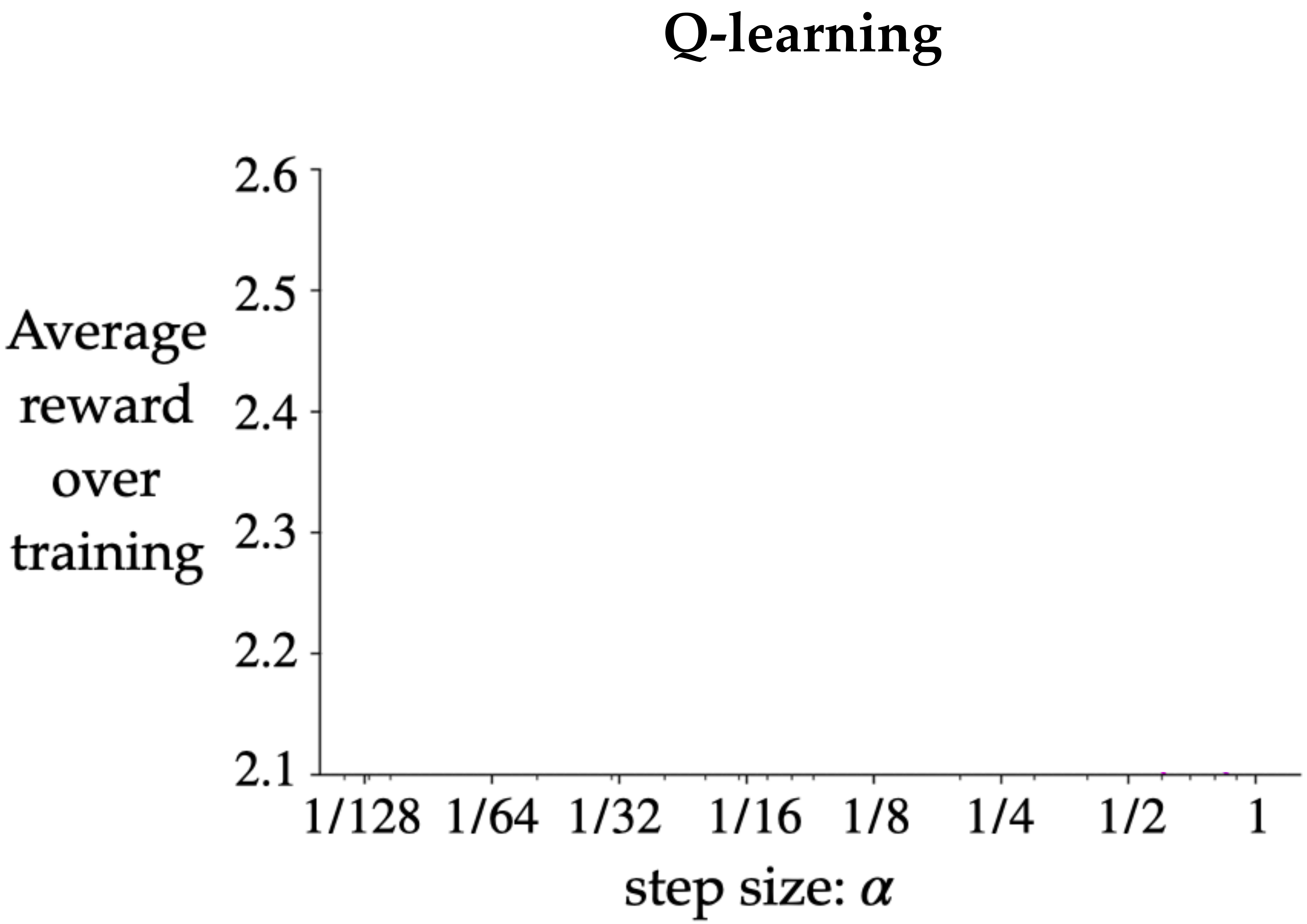

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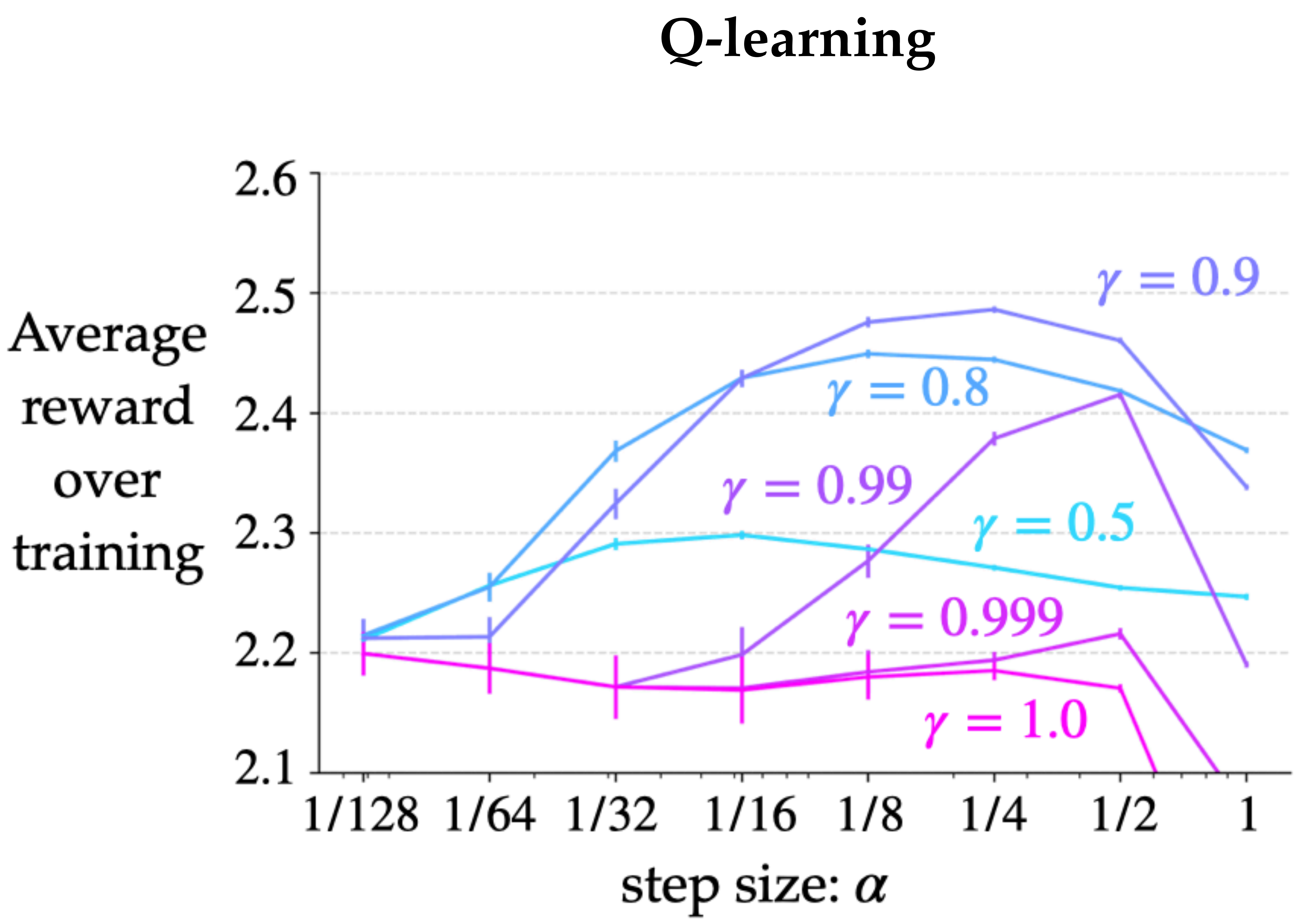

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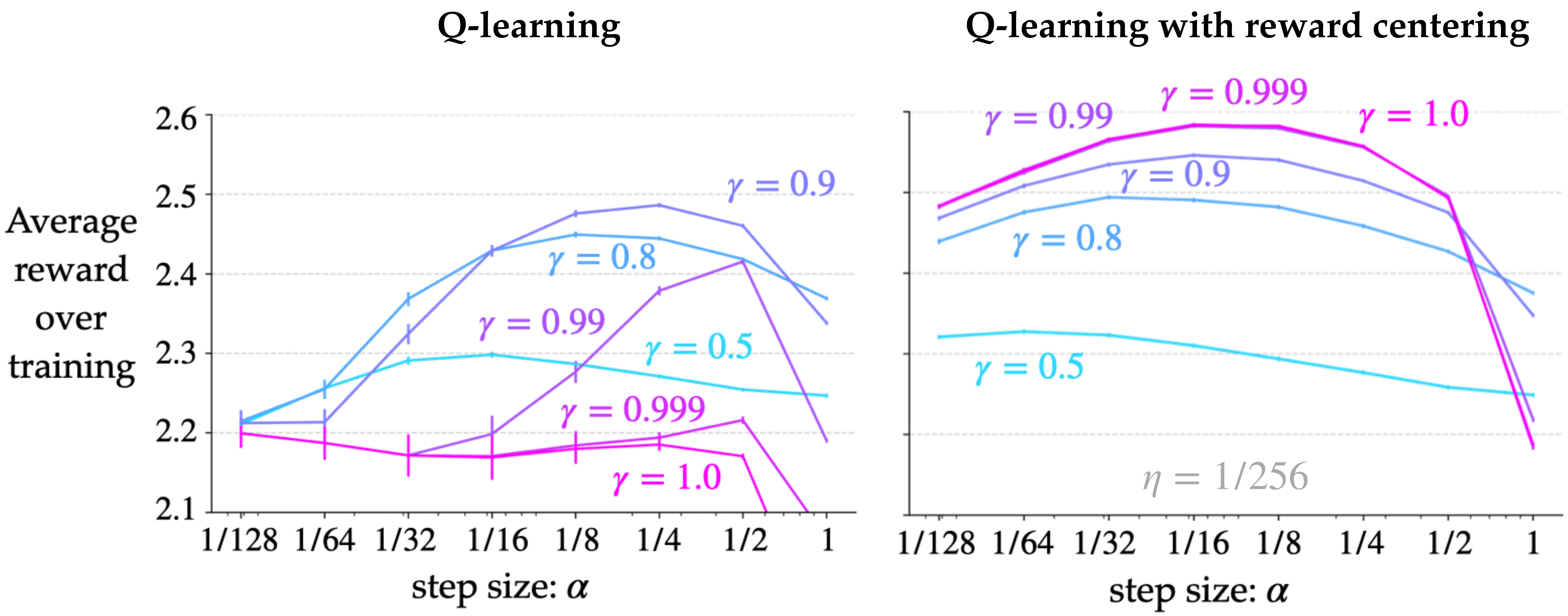

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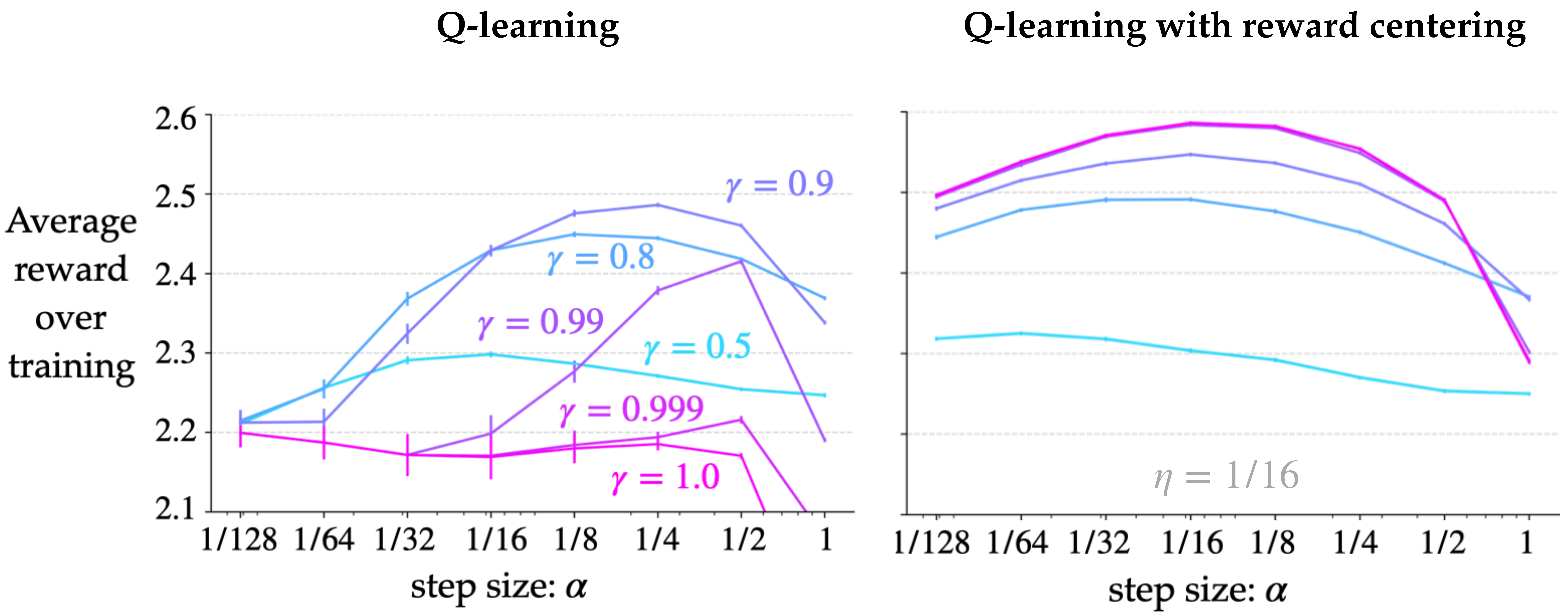

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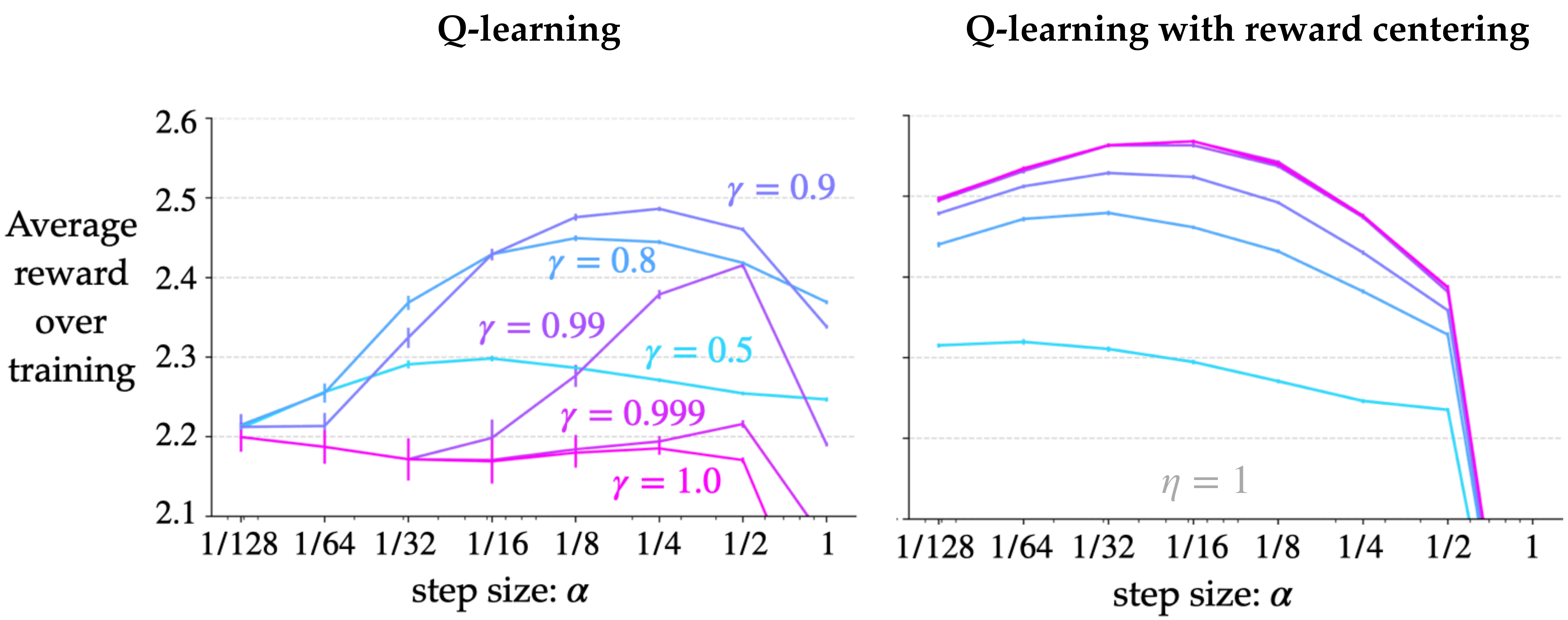

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KEY INSIGHT

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$$v_\pi^\gamma(s) \doteq \mathbb{E}_\pi\left[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots \mid S_t = s\right]$$

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$$\tilde{v}_\pi^\gamma(s) \doteq \mathbb{E}_\pi\Big[\sum_{k=0}^{\infty} \gamma^k \big(R_{t+k+1} - r(\pi)\big) \,|\, S_t = s\Big]$$

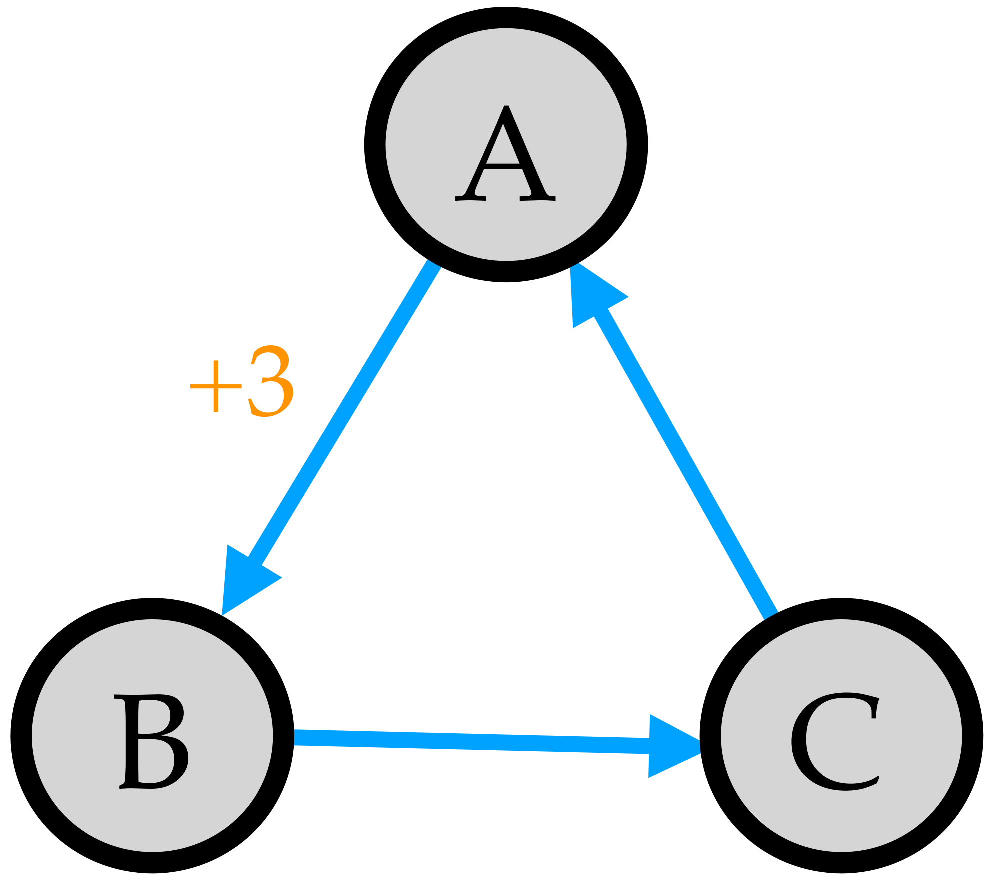
Centered discounted value function

MORE INTUITION

$$v_\pi^\gamma(s) = \frac{r(\pi)}{1-\gamma} + \tilde{v}_\pi(s) + e_\pi^\gamma(s)$$

$$\tilde{v}_\pi^\gamma(s)$$

MORE INTUITION



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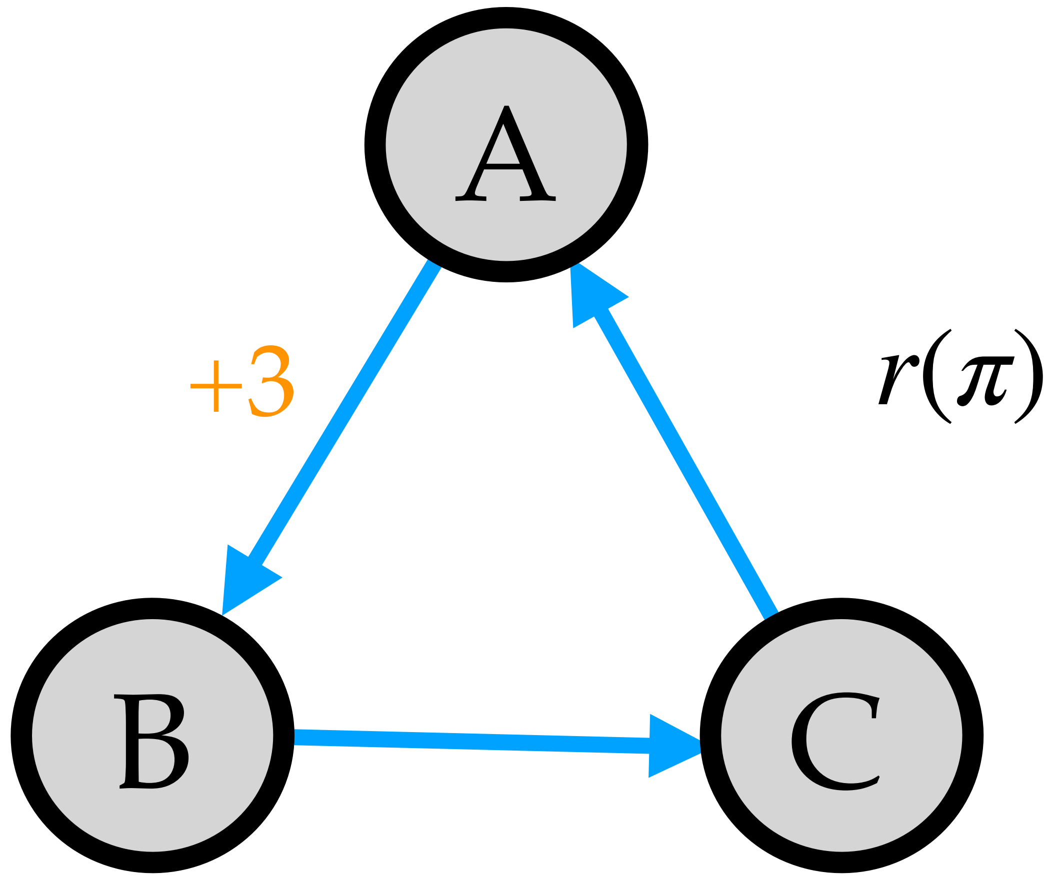
$\tilde{v}_\pi^\gamma(s)$

MORE INTUITION



$r(\pi) = 1$

$$v_\pi^\gamma(s) = \frac{r(\pi)}{1-\gamma} + \tilde{v}_\pi(s) + e_\pi^\gamma(s)$$

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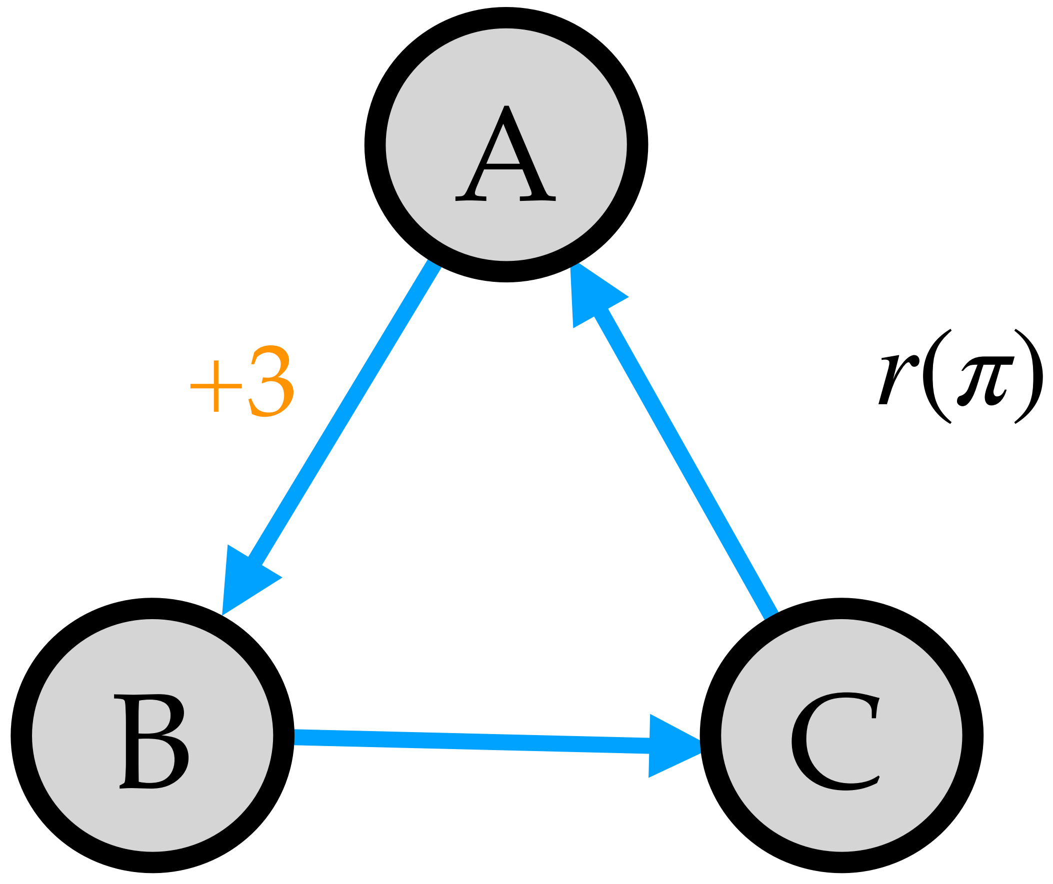
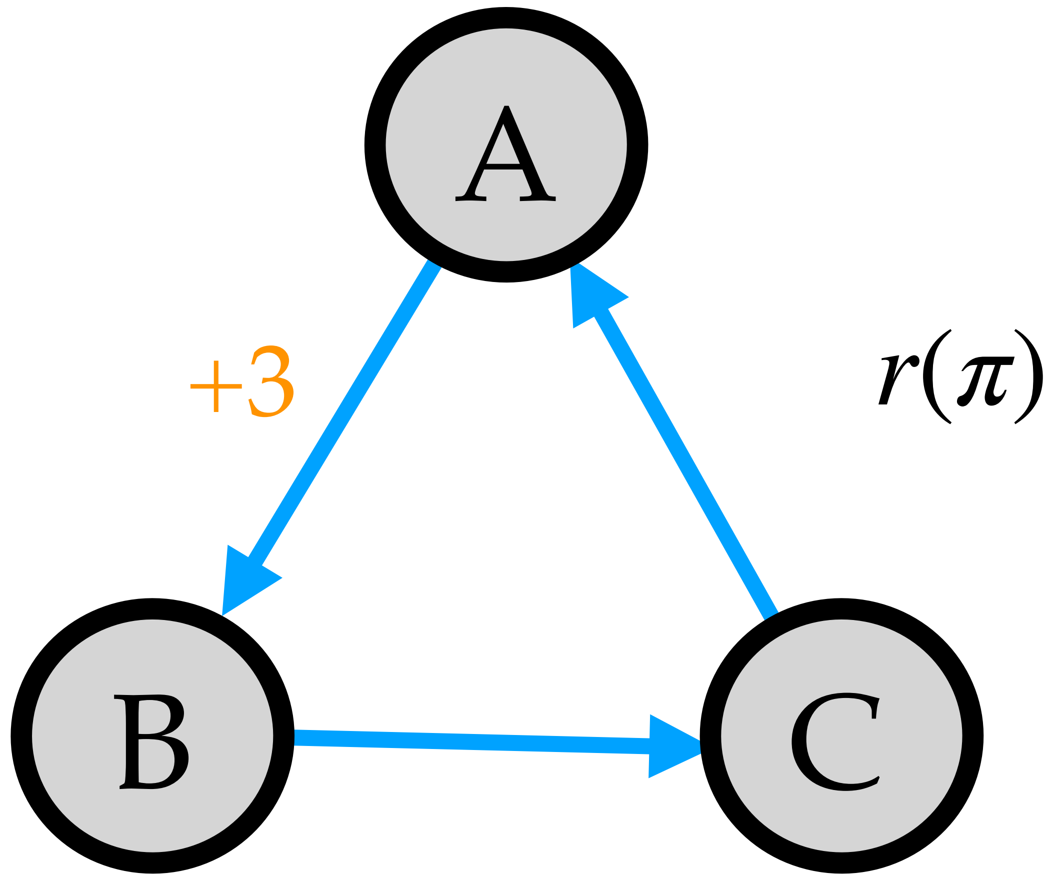


$$v_\pi^\gamma(s) = \frac{r(\pi)}{1-\gamma} + \tilde{v}_\pi(s) + e_\pi^\gamma(s)$$

$\tilde{v}_\pi^\gamma(s)$

	s_A	s_B	s_C
Standard discounted values			
Centered discounted values			
Differential values	1	-1	0

MORE INTUITION



	$\frac{r(\pi)}{1-\gamma}$
$\gamma = 0.8$	5

	s_A	s_B	s_C
Standard discounted values			
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$$v_\pi^\gamma(s) = \frac{r(\pi)}{1-\gamma} + \tilde{v}_\pi(s) + e_\pi^\gamma(s)$$

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MORE INTUITION

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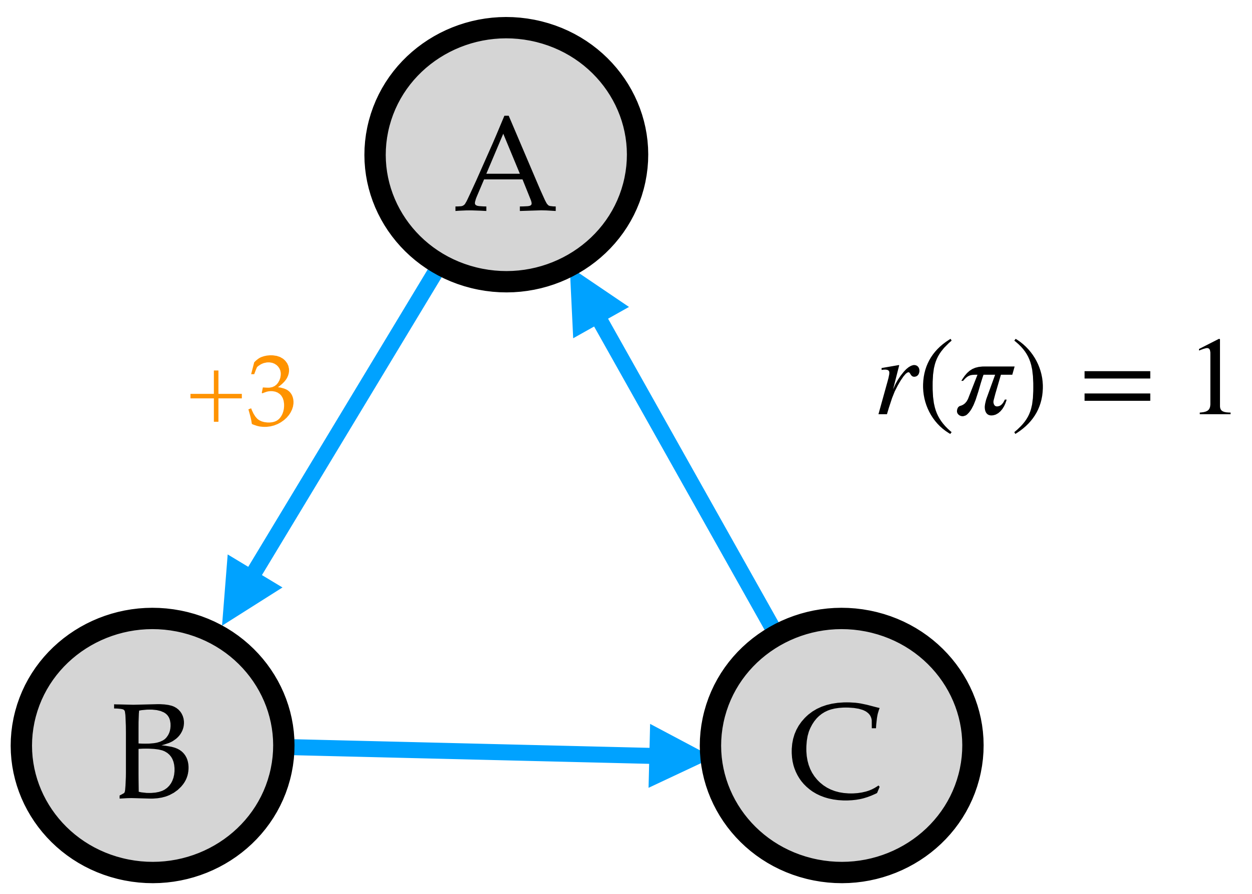
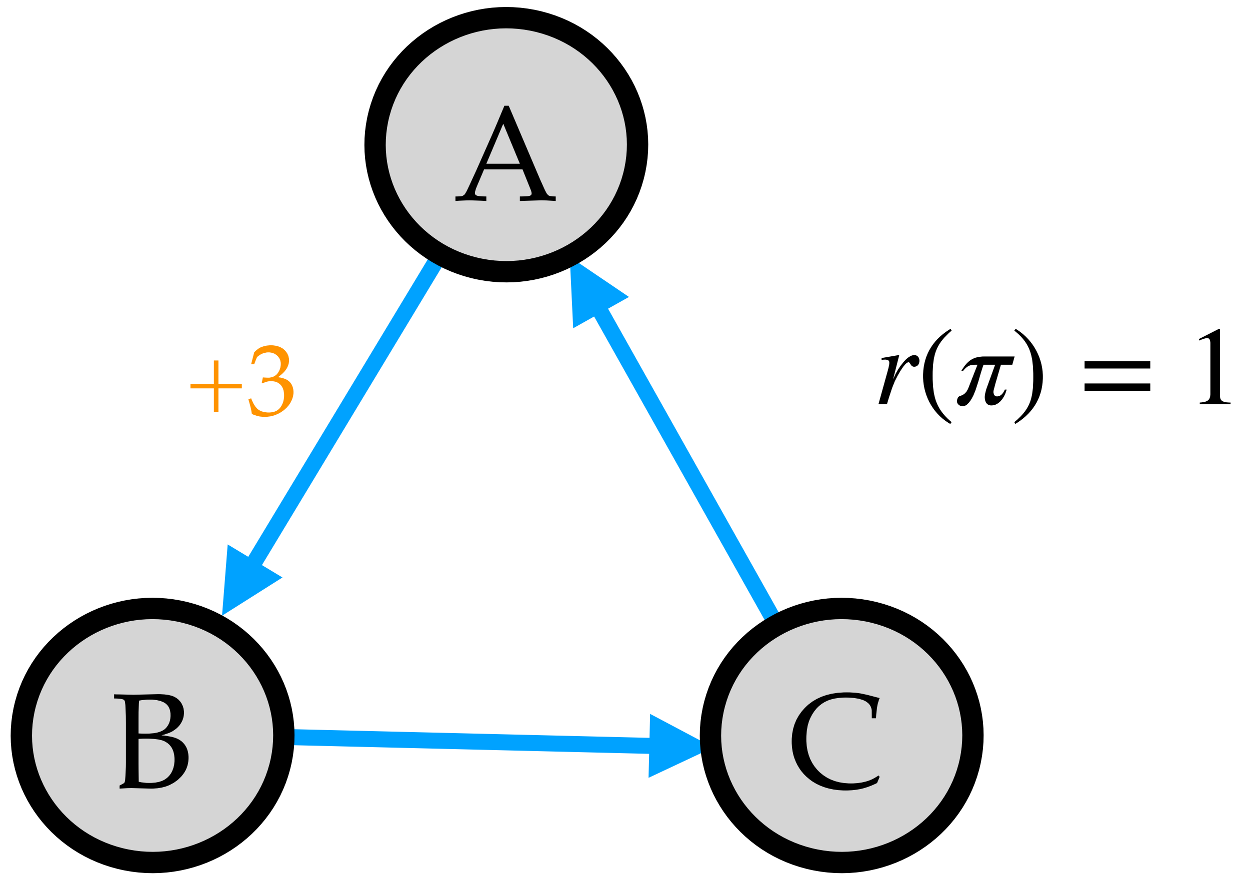


		s_A	s_B	s_C
Standard discounted values	$\gamma = 0.8$	6.15	3.93	4.92
Centered discounted values	$\gamma = 0.8$	1.15	-1.07	-0.08
Differential values		1	-1	0

$$v_\pi^\gamma(s) = \frac{r(\pi)}{1-\gamma} + \underbrace{\tilde{v}_\pi(s) + e_\pi^\gamma(s)}_{\tilde{v}_\pi^\gamma(s)}$$

MORE INTUITION



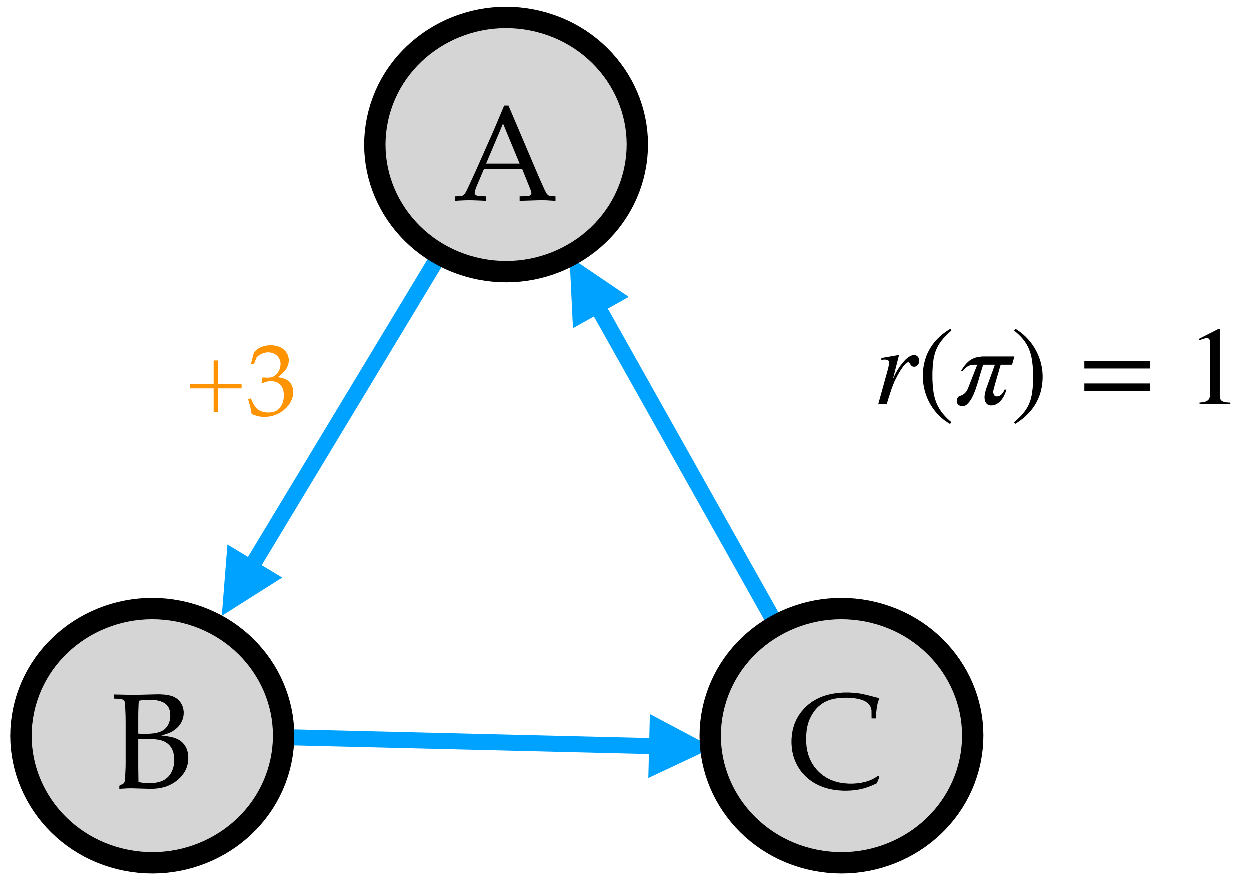
	$\frac{r(\pi)}{1-\gamma}$
$\gamma = 0.8$	5
$\gamma = 0.9$	10

		s_A	s_B	s_C
Standard discounted values	$\gamma = 0.8$	6.15	3.93	4.92
	$\gamma = 0.9$	11.07	8.97	9.96
Centered discounted values	$\gamma = 0.8$	1.15	-1.07	-0.08
	$\gamma = 0.9$	1.07	-1.03	-0.04
Differential values		1	-1	0

$$v_\pi^\gamma(s) = \frac{r(\pi)}{1-\gamma} + \underbrace{\tilde{v}_\pi(s) + e_\pi^\gamma(s)}_{\tilde{v}_\pi^\gamma(s)}$$

MORE INTUITION

	$\frac{r(\pi)}{1-\gamma}$
$\gamma = 0.8$	5
$\gamma = 0.9$	10
$\gamma = 0.99$	100

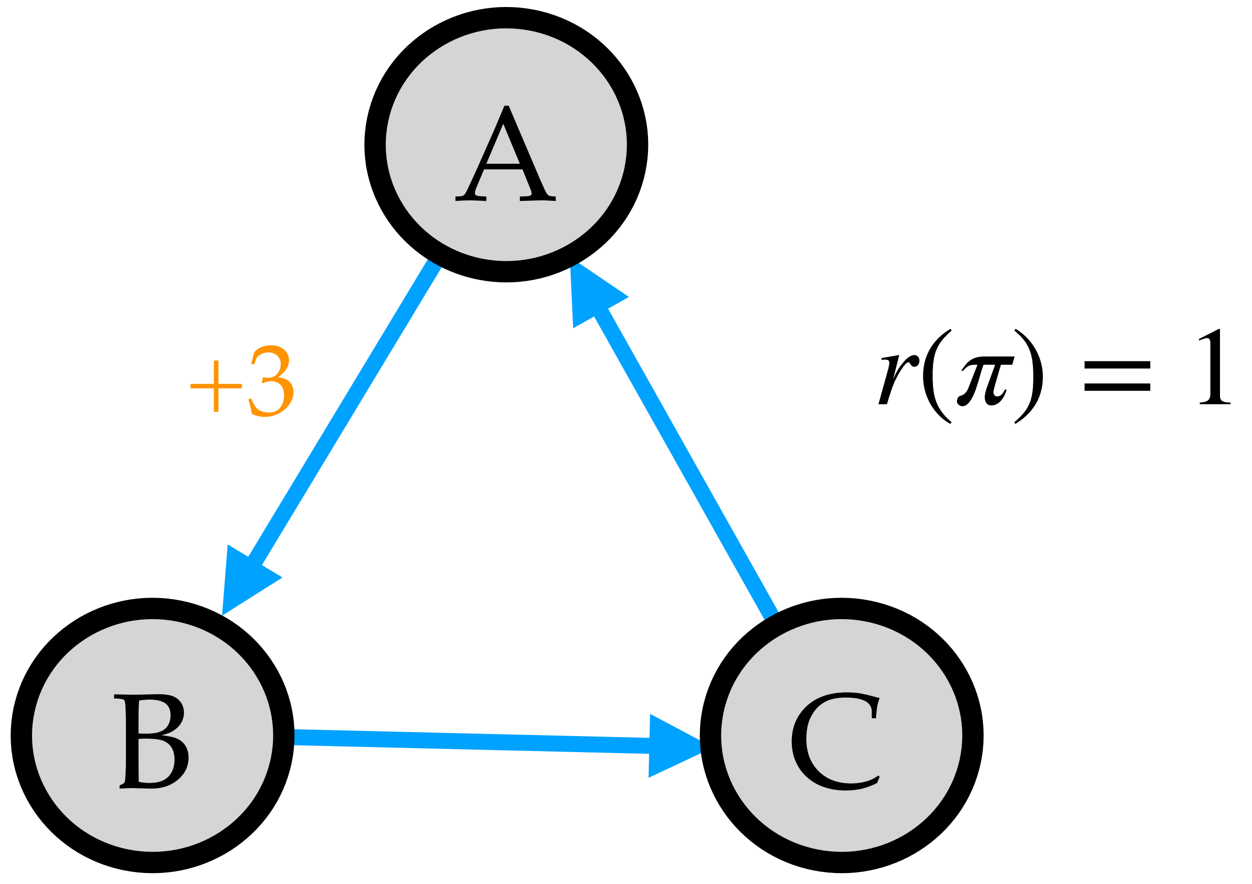


$$v_\pi^\gamma(s) = \frac{r(\pi)}{1-\gamma} + \tilde{v}_\pi(s) + e_\pi^\gamma(s)$$

$\tilde{v}_\pi^\gamma(s)$

		s_A	s_B	s_C
Standard discounted values	$\gamma = 0.8$	6.15	3.93	4.92
	$\gamma = 0.9$	11.07	8.97	9.96
	$\gamma = 0.99$	101.01	98.99	99.99
Centered discounted values	$\gamma = 0.8$	1.15	-1.07	-0.08
	$\gamma = 0.9$	1.07	-1.03	-0.04
	$\gamma = 0.99$	1.01	-1.01	-0.01
Differential values		1	-1	0

MORE INTUITION



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ESTIMATING $r(\pi)$

$$S_0 \; A_0 \; R_1 \; S_1 \; A_1, R_2 \ldots \; S_t \; A_t \; R_{t+1} \; S_{t+1} \; A_{t+1} \; R_{t+2} \; \ldots$$

ESTIMATING $r(\pi)$

$$S_0 \; A_0 \; R_1 \; S_1 \; A_1, R_2 \ldots \; S_t \; A_t \; R_{t+1} \; S_{t+1} \; A_{t+1} \; R_{t+2} \; \ldots$$

On-policy

ESTIMATING $r(\pi)$

$$S_0 \ A_0 \ R_1 \ S_1 \ A_1, R_2 \dots \ S_t \ A_t \ R_{t+1} \ S_{t+1} \ A_{t+1} \ R_{t+2} \ \dots$$

On-policy

$$\bar{R}_{t+1} \doteq \bar{R}_t + \beta_t (R_{t+1} - \bar{R}_t)$$

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$$S_0 \; A_0 \; R_1 \; S_1 \; A_1, R_2 \ldots \; S_t \; A_t \; R_{t+1} \; S_{t+1} \; A_{t+1} \; R_{t+2} \; \ldots$$

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where $\delta_t \doteq R_{t+1} - \bar{R}_t + \gamma V_t(S_{t+1}) - V_t(S_t)$

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$$S_0\ A_0\ R_1\ S_1\ A_1, R_2 \ldots\ S_t\ A_t\ R_{t+1}\ S_{t+1}\ A_{t+1}\ R_{t+2}\ \ldots$$

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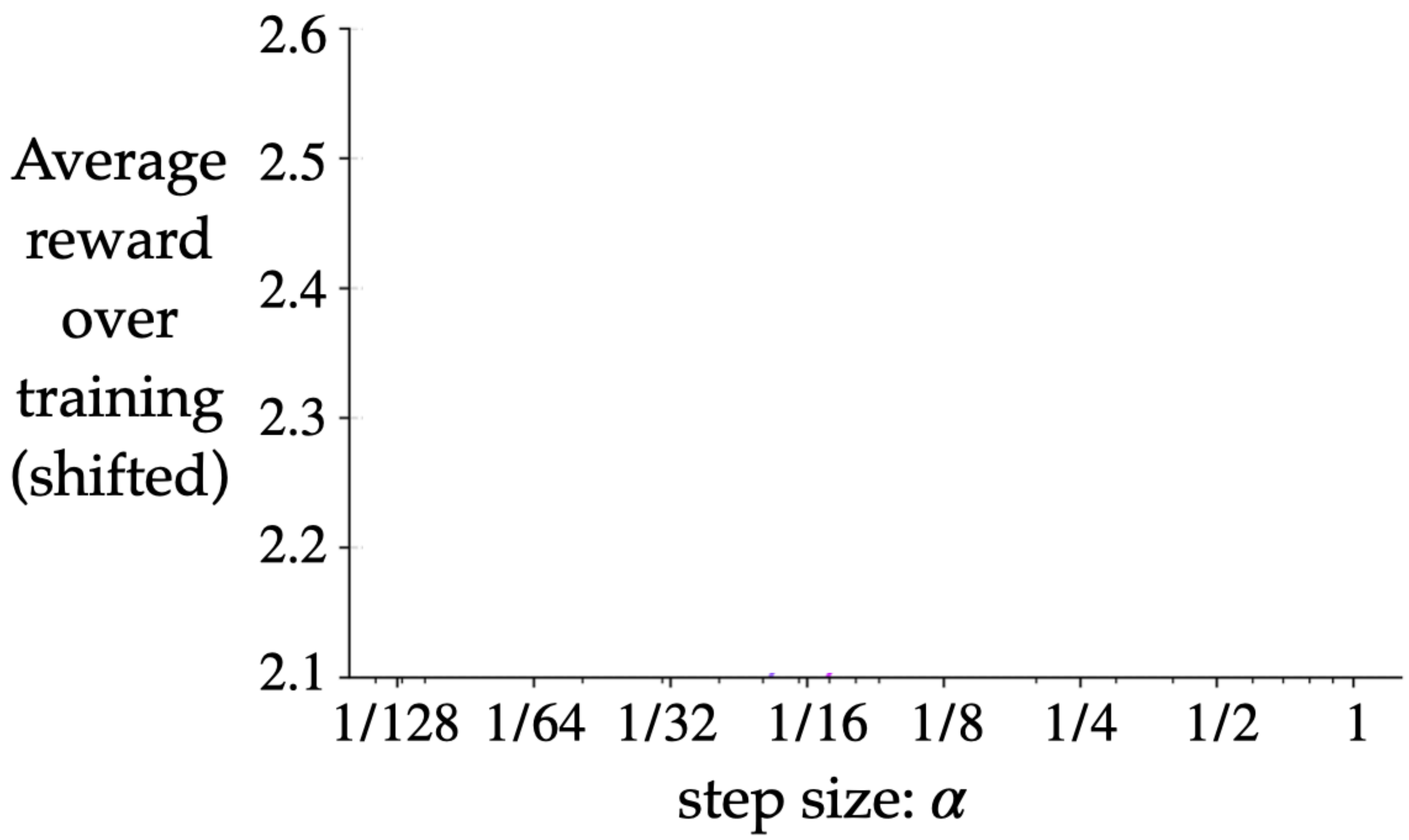
MORE ROBUST TO SHIFTED REWARDS



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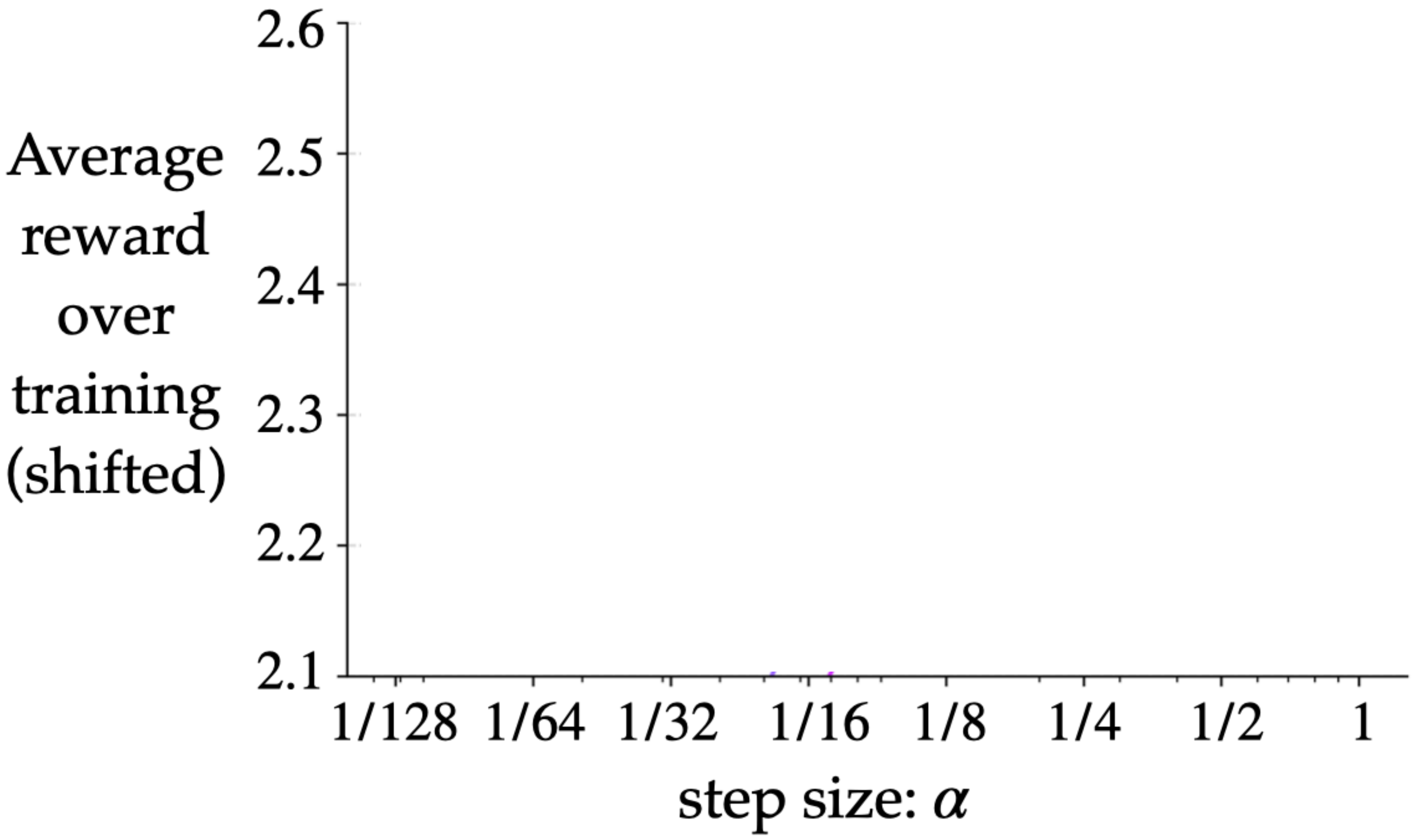

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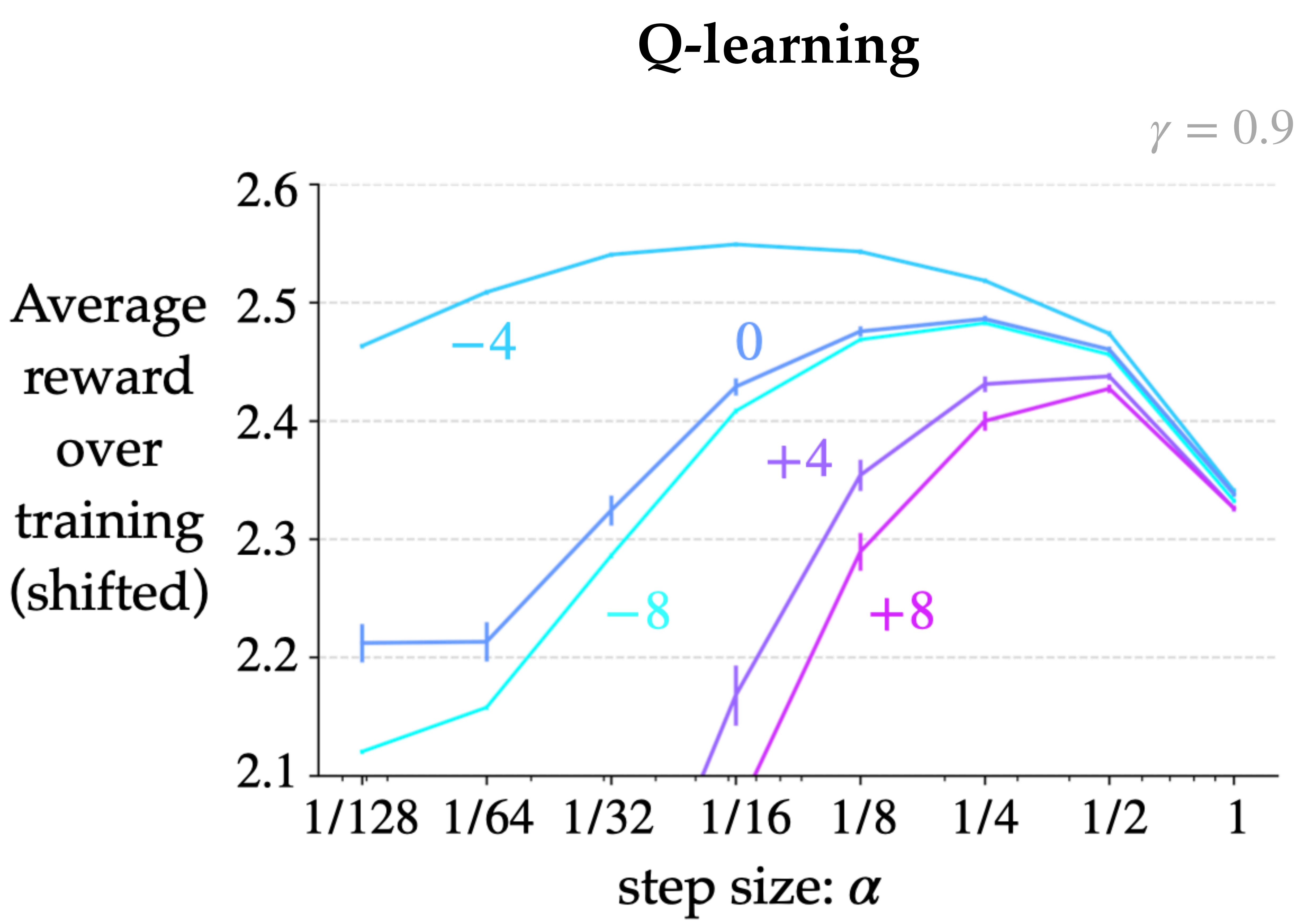

AccessControl (tabular)

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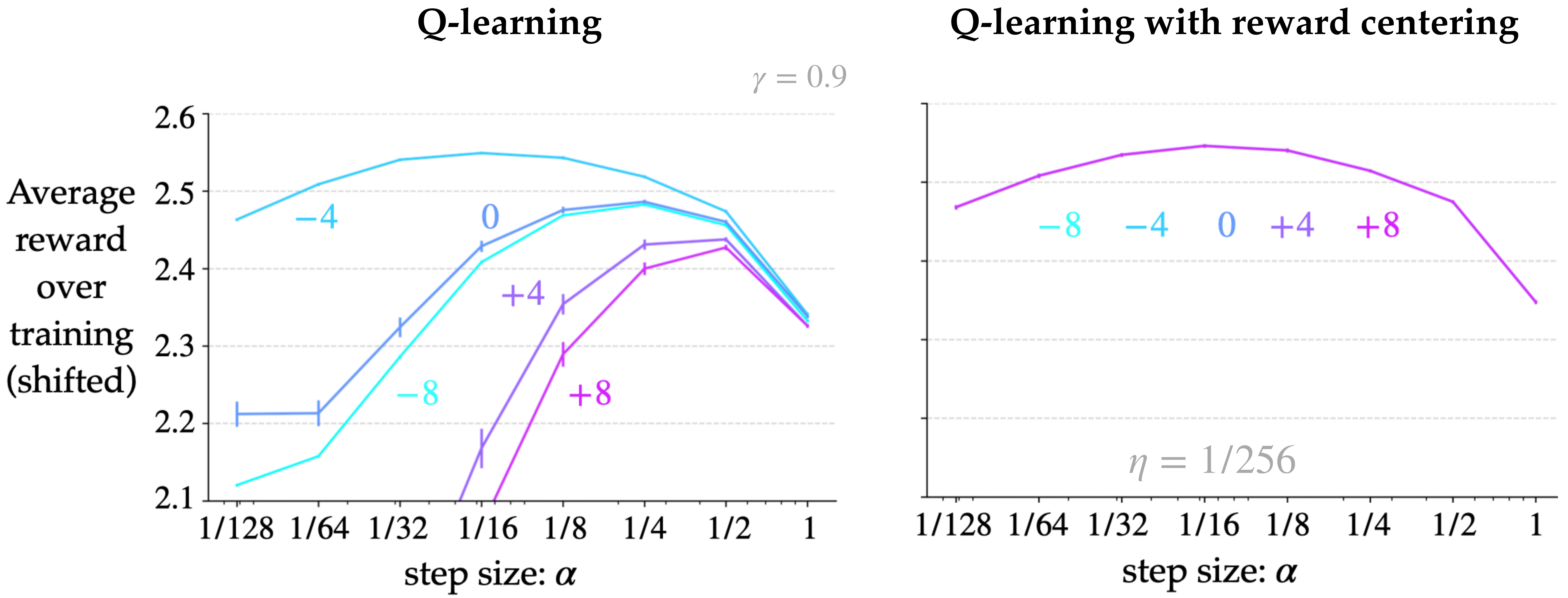

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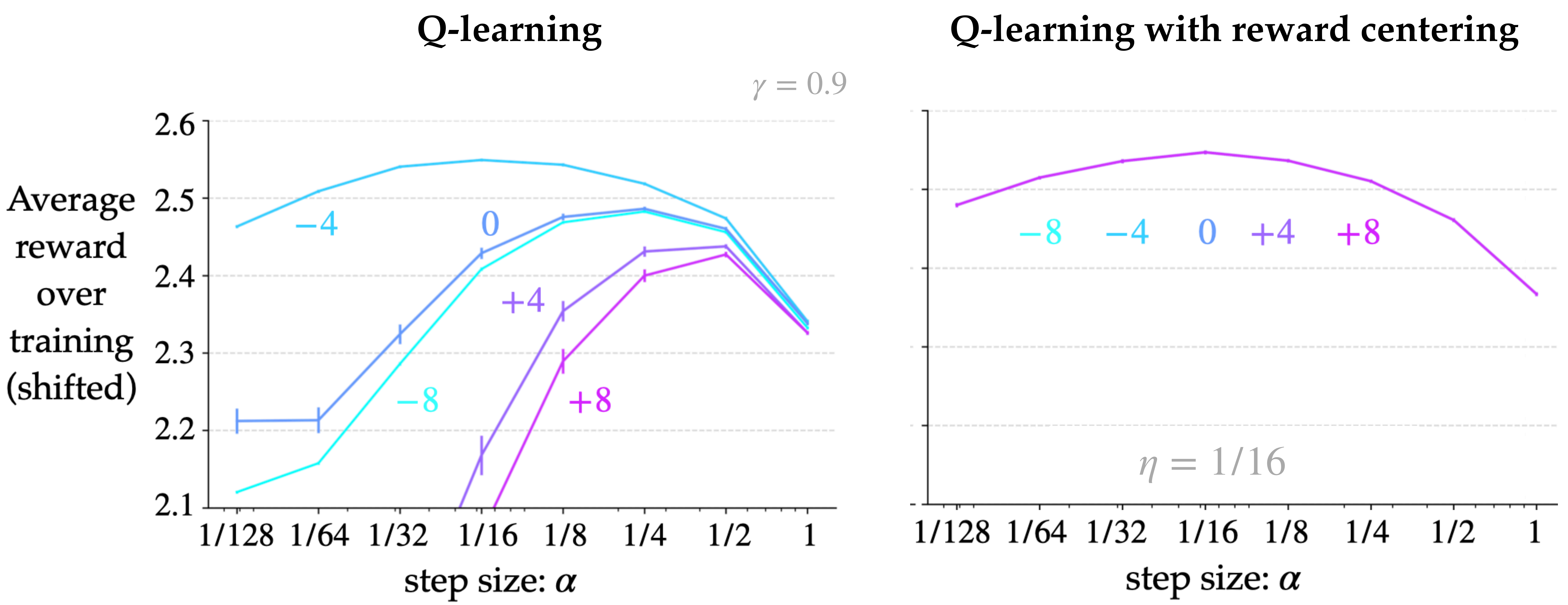

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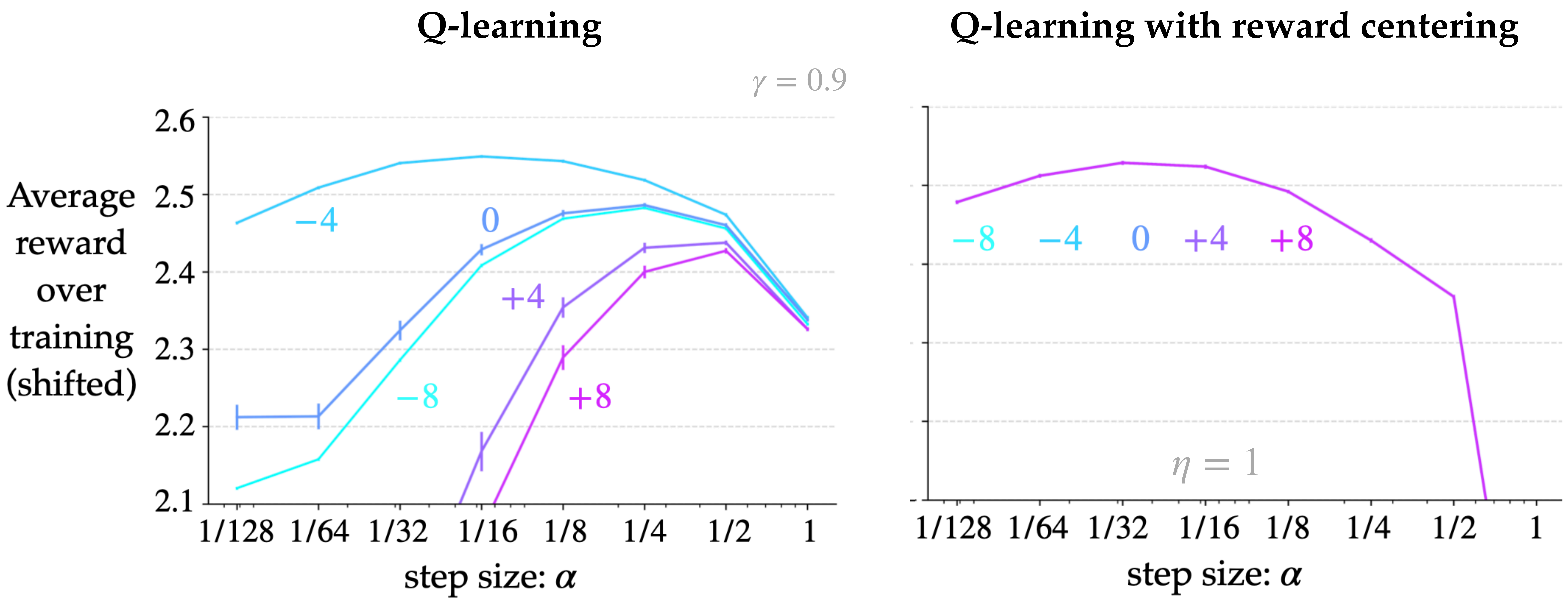

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- Unlocks algorithms in which the discount factor can be efficiently adapted over time

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OUTLINE

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2. *Multi-step* average-reward methods
3. An idea to improve *discounted-reward* methods

Conclusions, limitations, and future work

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"To develop simple and practical learning algorithms from first principles for long-lived agents"

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ACKNOWLEDGMENTS

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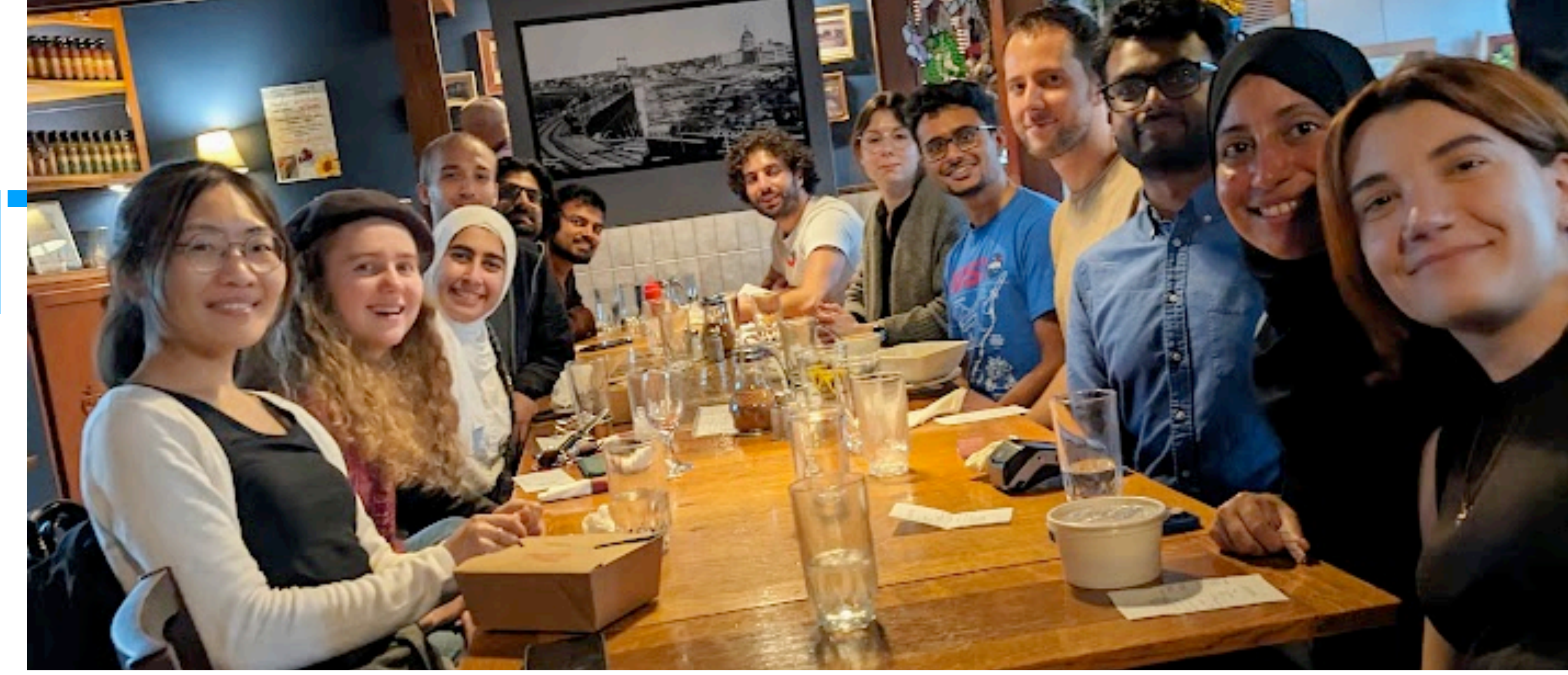

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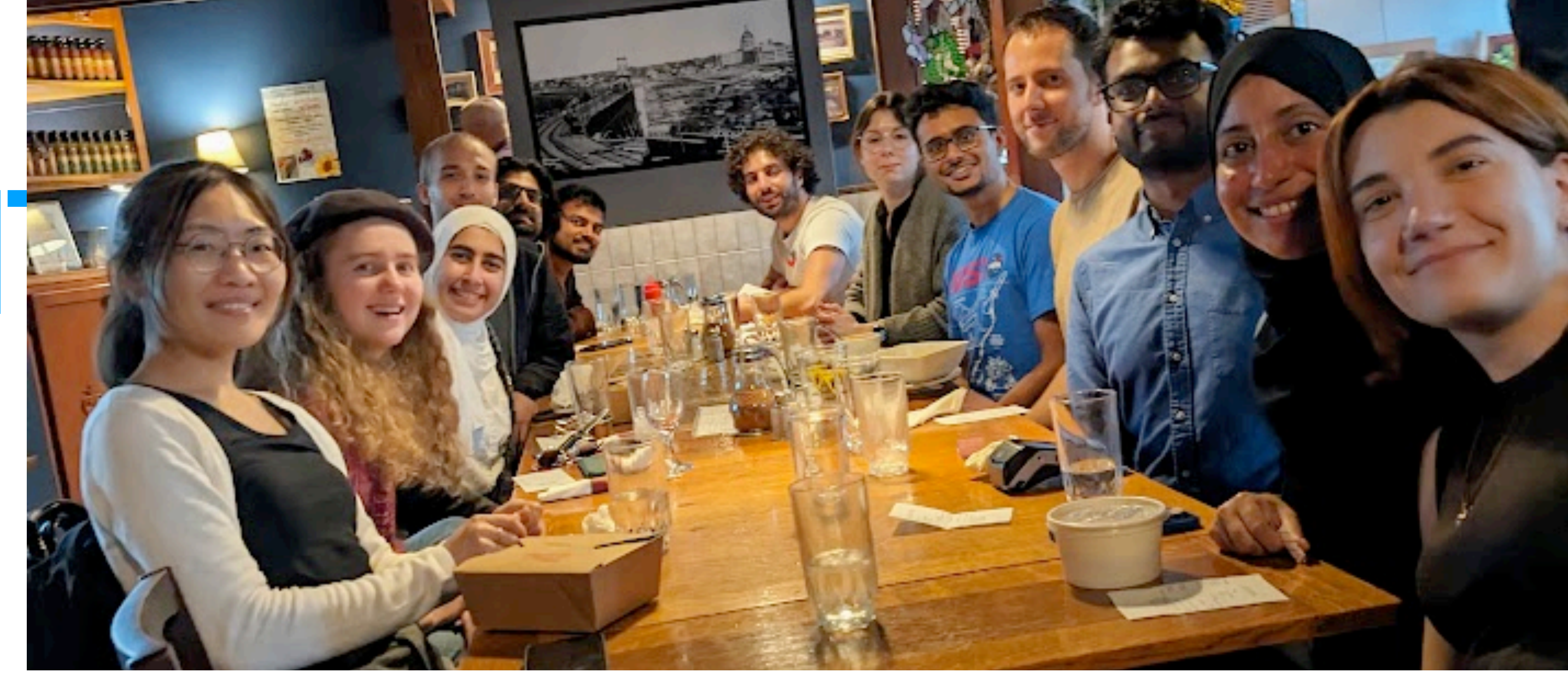

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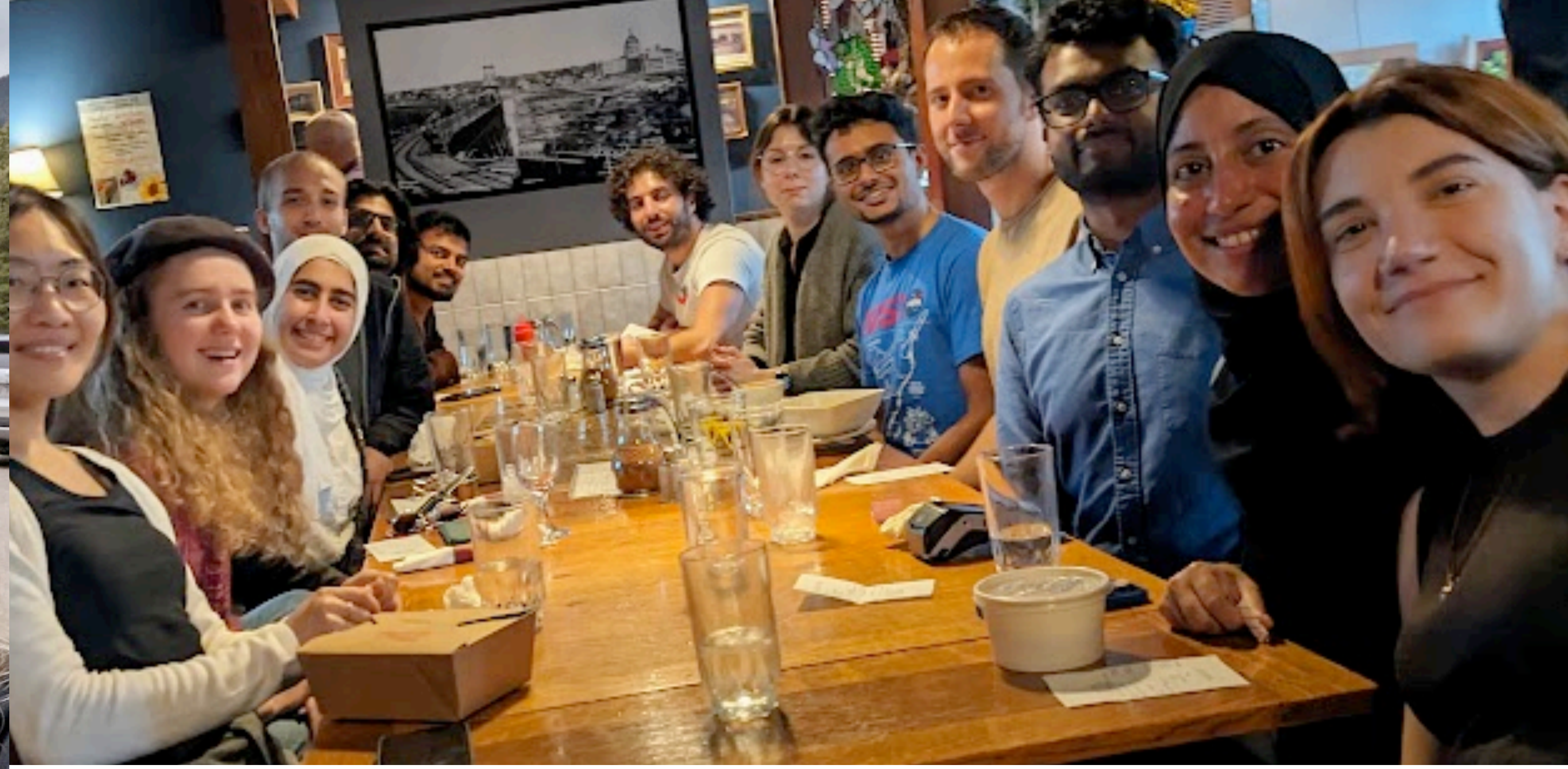

CANADA



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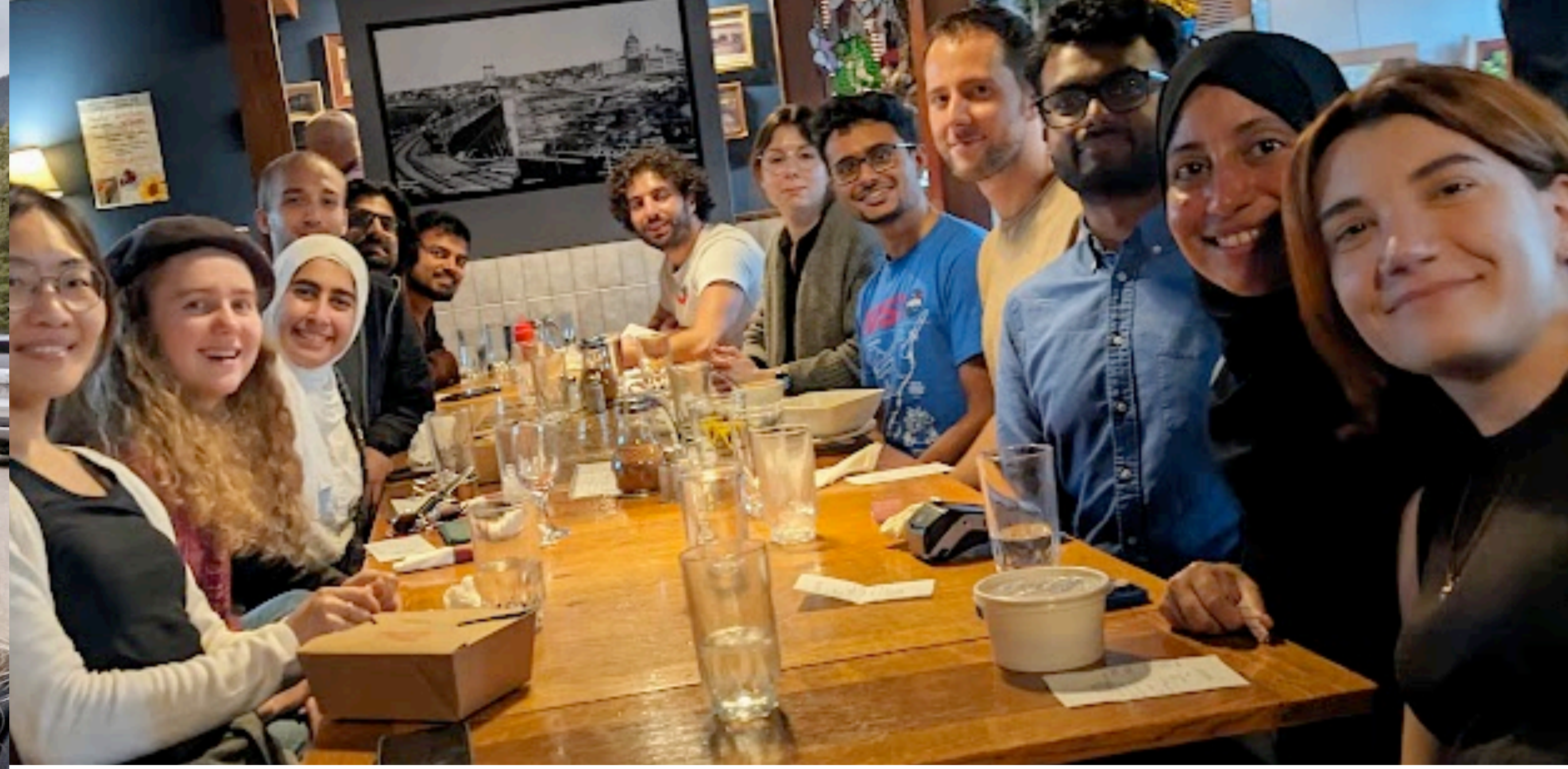


CANADA


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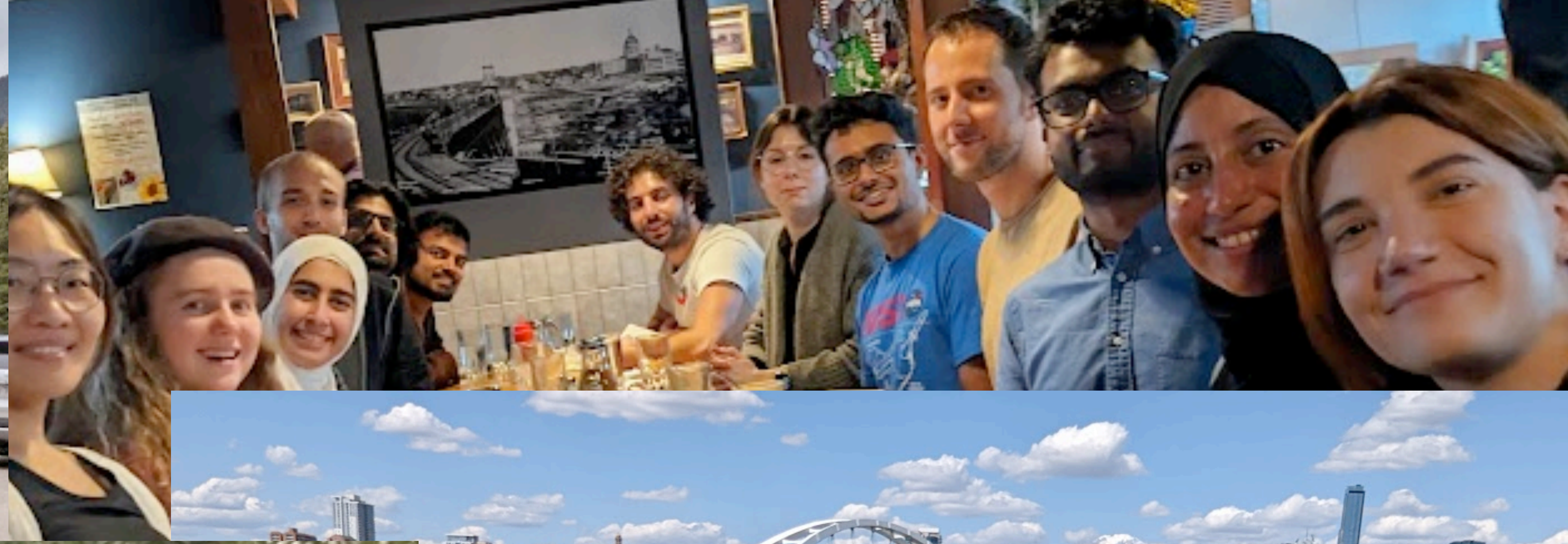
MOOSEHEAD






Wilson


CANADA









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MOOSEHEAD

TRY HARDS
CCM
CANADA


TRY HARDS
CANADA


CANADA


CANADA


TRY HARDS
CANADA
Urban Riders


CANADA

THANK YOU

Questions?