ESSENTIALS OF RL

Reinforcement Learning: Lecture 2

3rd Nepal Winter School in AI

24th Dec 2021

Abhishek Naik

OUTLINE

- Dynamic Programming (DP)
- Temporal-Difference (TD) Learning
- Model-based RL
- Policy Optimization

REINFORCEMENT LEARNING

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- Goal: learning some behaviour to maximize a numerical reward signal

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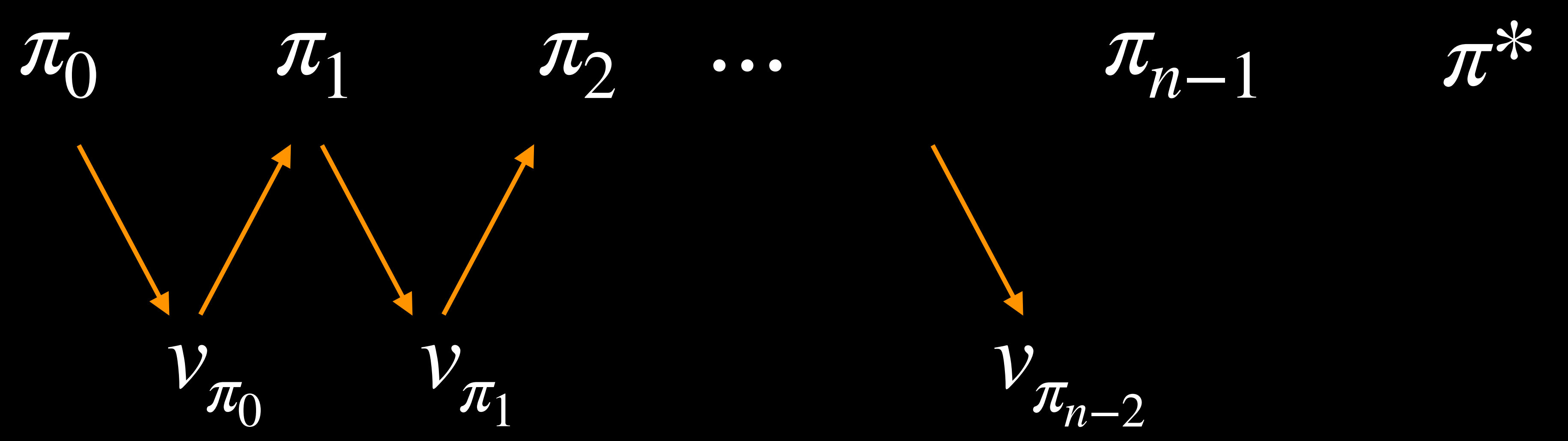

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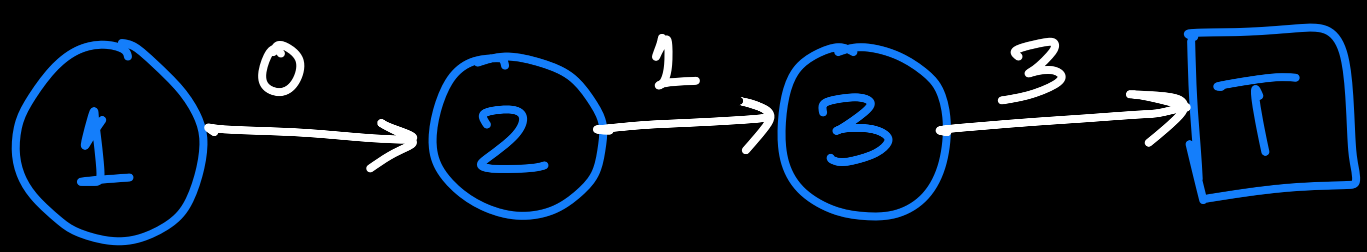

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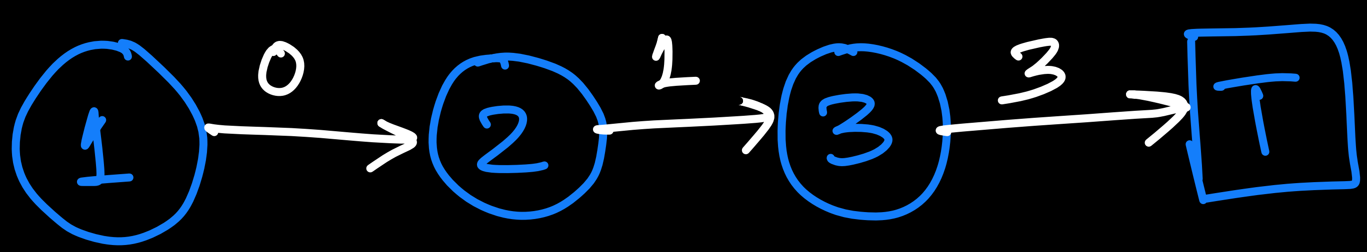

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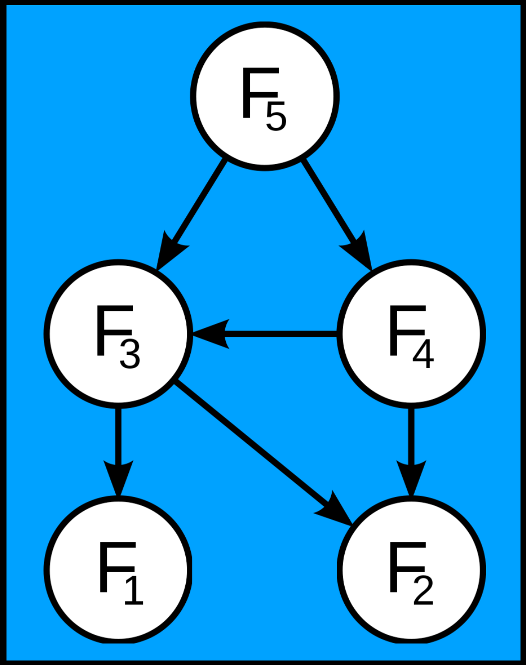

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$$S_0, A_0, R_1, S_1, \ldots, S_t, A_t, R_{t+1}, S_{t+1}, \ldots$$

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&= \bar{x}_N + \frac{1}{N+1}(x_{N+1} - \bar{x}_N)
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&= \bar{x}_N + \frac{1}{N+1}(x_{N+1} - \bar{x}_N) \\
\bar{x}_{N+1} &= \bar{x}_N + \alpha(x_{N+1} - \bar{x}_N)
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$$

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DP $$v_{t+1}(s) = \sum_a \pi(a \mid s) \sum_{s',r} p(s', r \mid s, a) \left[r + v_t(s') \right]$$

IMPROVING ESTIMATES FROM A STREAM OF DATA

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TEMPORAL-DIFFERENCE (TD) LEARNING

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Algorithm : Tabular TD learning to estimate v_π

Input: The target policy π

Algorithm parameters: step size $\alpha \in (0, 1]$

Initialize $V(s)$, for all $s \in \mathcal{S}$, arbitrarily (e.g., to zero)

Observe initial state S

for *each time step* **do**

  $A \leftarrow$ action according to π in S

  Take action A, observe R, S'

  $V(S) \leftarrow V(S) + \alpha \left[R + V(S') - V(S)\right]$

  $S \leftarrow S'$

end

return V

CONTROL: EXPLORATION VS EXPLOITATION

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  - with a small probability, pick a random action

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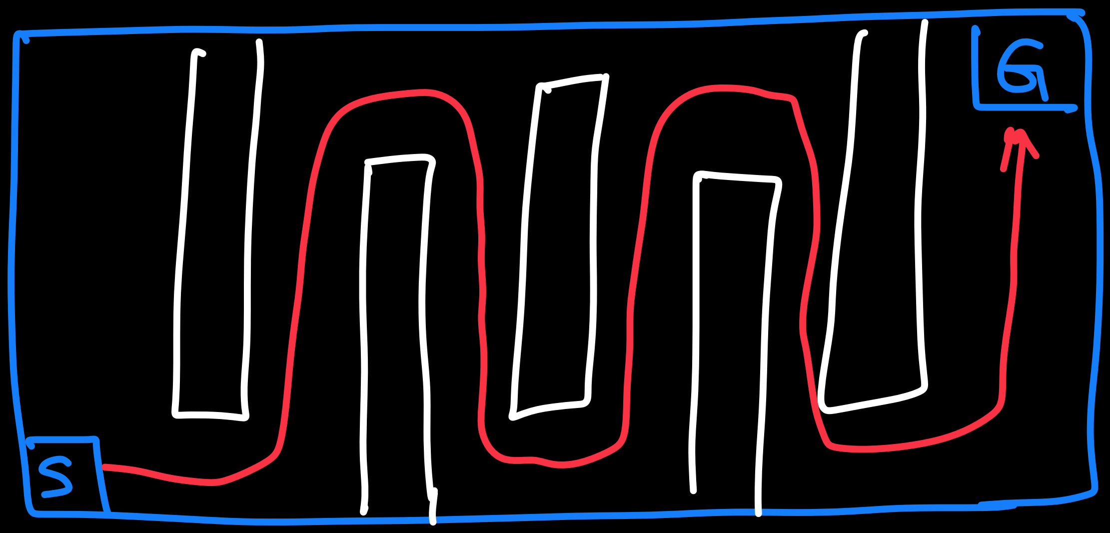

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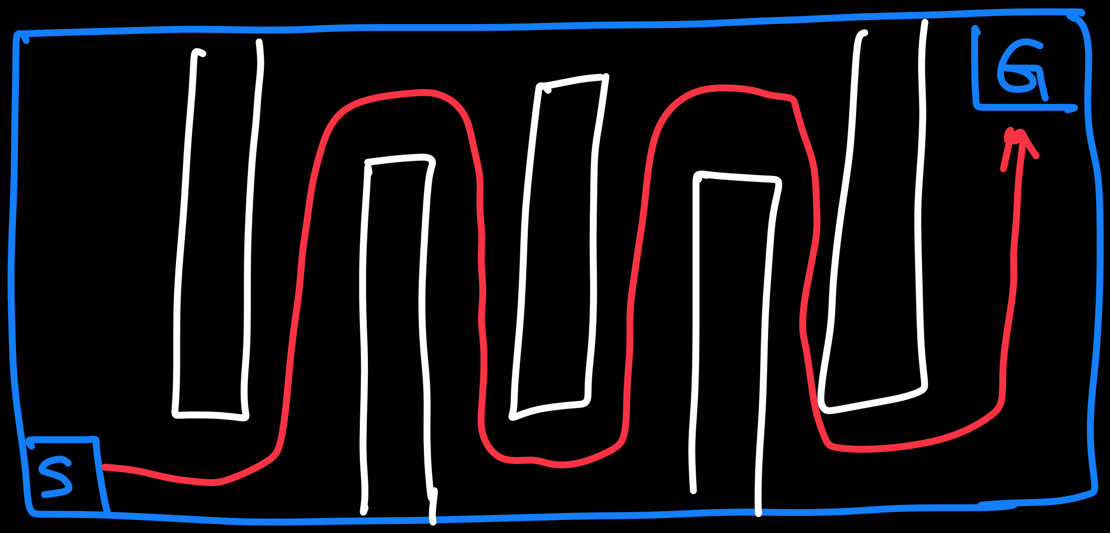

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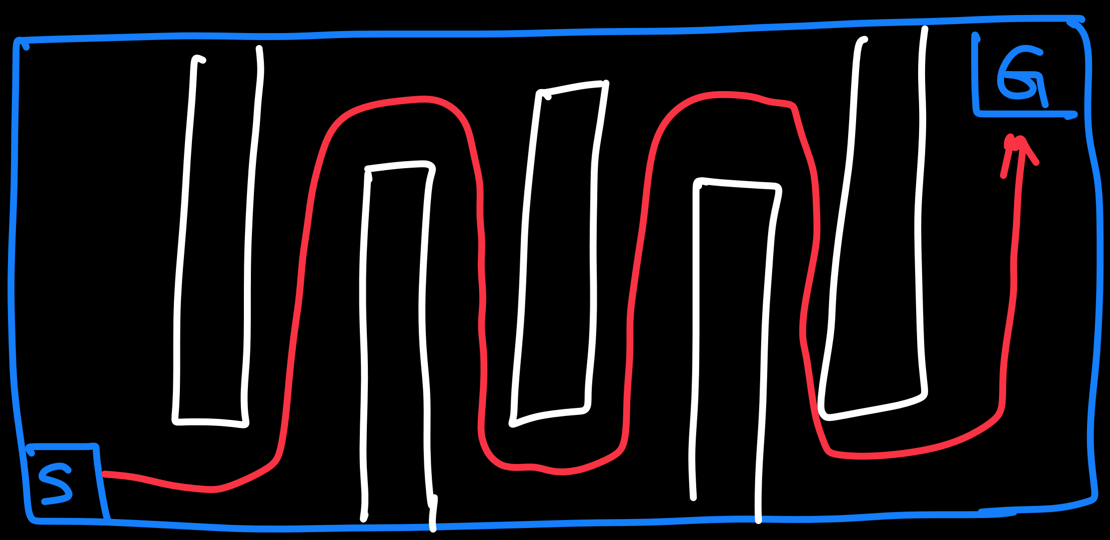

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ϵ-greedy action selection

CONTROL ALGORITHM: SARSA

Algorithm : SARSA to estimate $Q \approx Q_{\pi^*}$

Parameters: step size $\alpha \in (0, 1]$

Initialize $Q(s, a)$, for all $s \in \mathcal{S}$, $a \in \mathcal{A}$, arbitrarily (e.g., to zero)
Observe initial state S
for *each time step* **do**
    $A \leftarrow$ action in S according to policy derived from Q (e.g., ϵ-greedy)
    Take action A, observe R, S'
    $A' \leftarrow$ action in S' according to policy derived from Q (e.g., ϵ-greedy)
    $Q(S, A) \leftarrow Q(S, A) + \alpha \left[R + Q(S', A') - Q(S, A)\right]$
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"On-policy"

CONTROL ALGORITHM: Q-LEARNING

Algorithm : Q-learning to estimate $Q \approx Q_{\pi^*}$

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  Take action A, observe R, S'

  $A' \leftarrow$ action in S' according to the greedy policy derived from Q

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"Off-policy"

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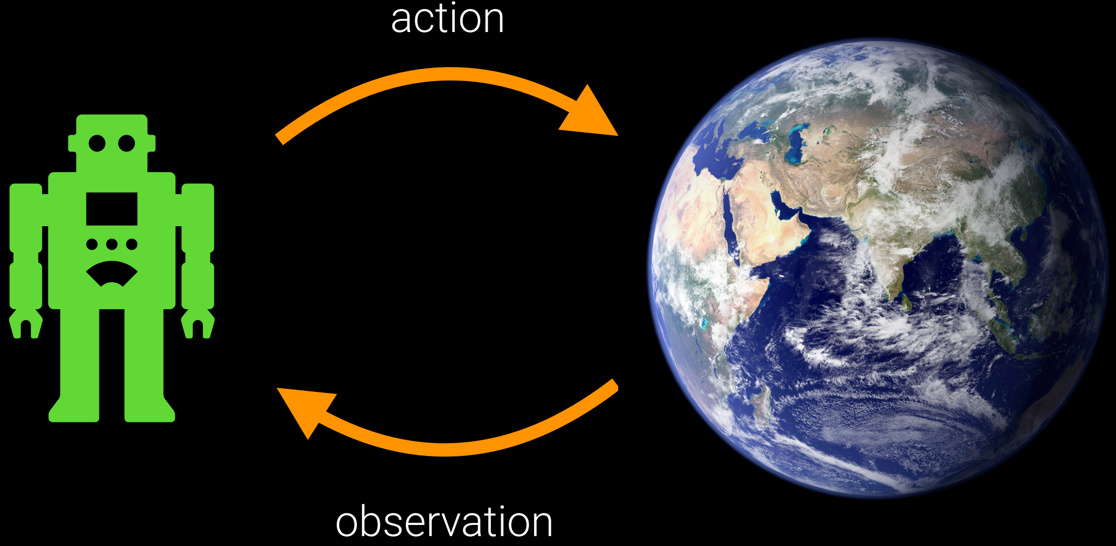

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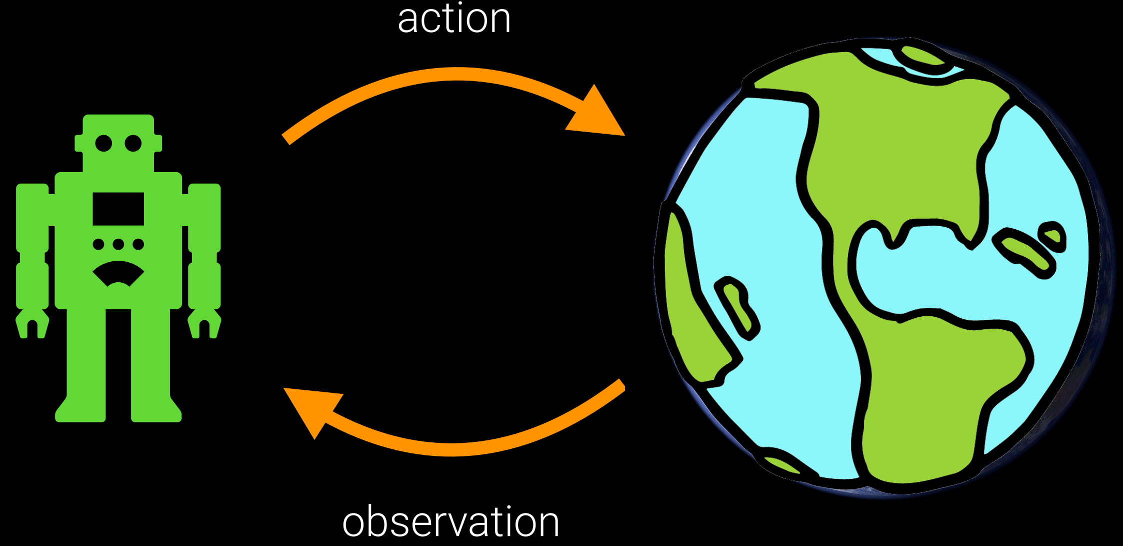

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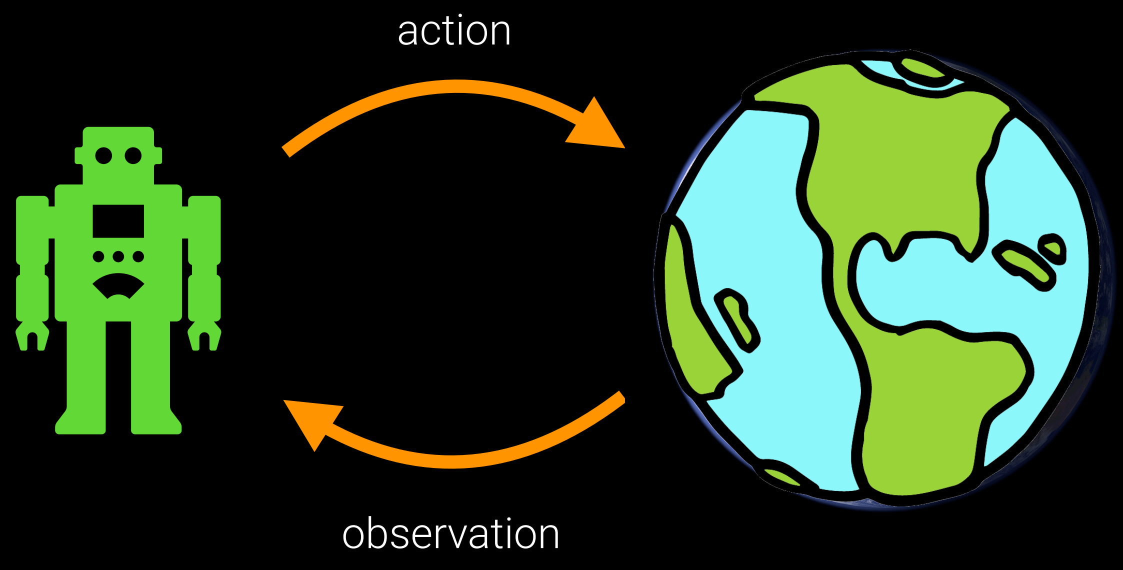

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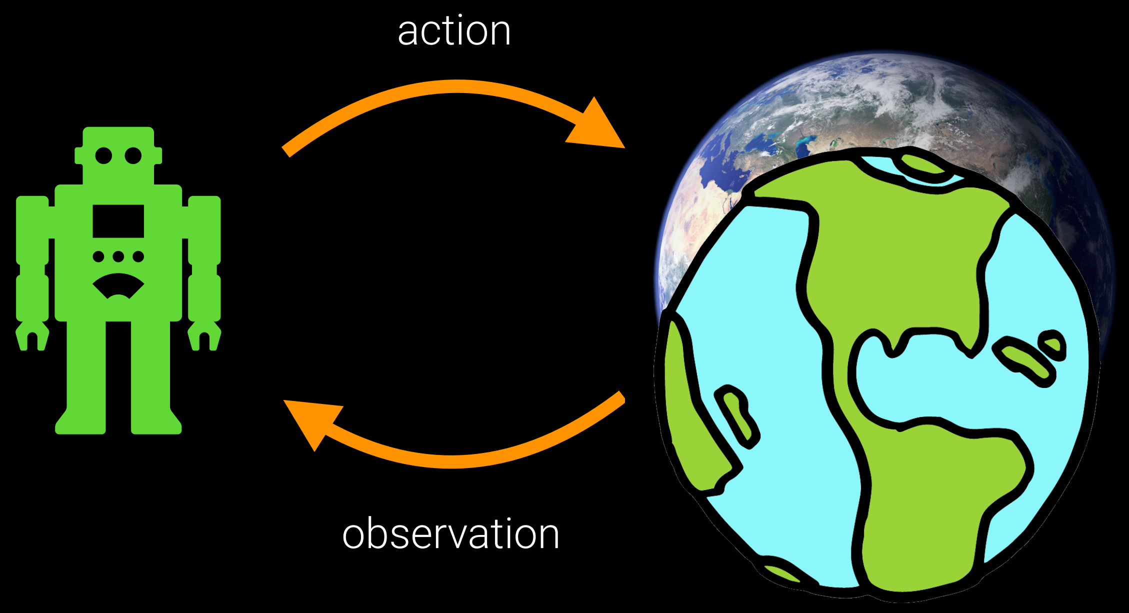

LEARNING A MODEL FROM EXPERIENCE

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DYNA: INTEGRATING LEARNING AND PLANNING

Learning the model

Planning with
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Algorithm : Dyna to estimate $Q \approx Q_{\pi^*}$

Parameters: step size $\alpha \in (0, 1]$

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    while *time to plan* **do**
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Planning with the learned model

OUTLINE

- Dynamic Programming (DP)
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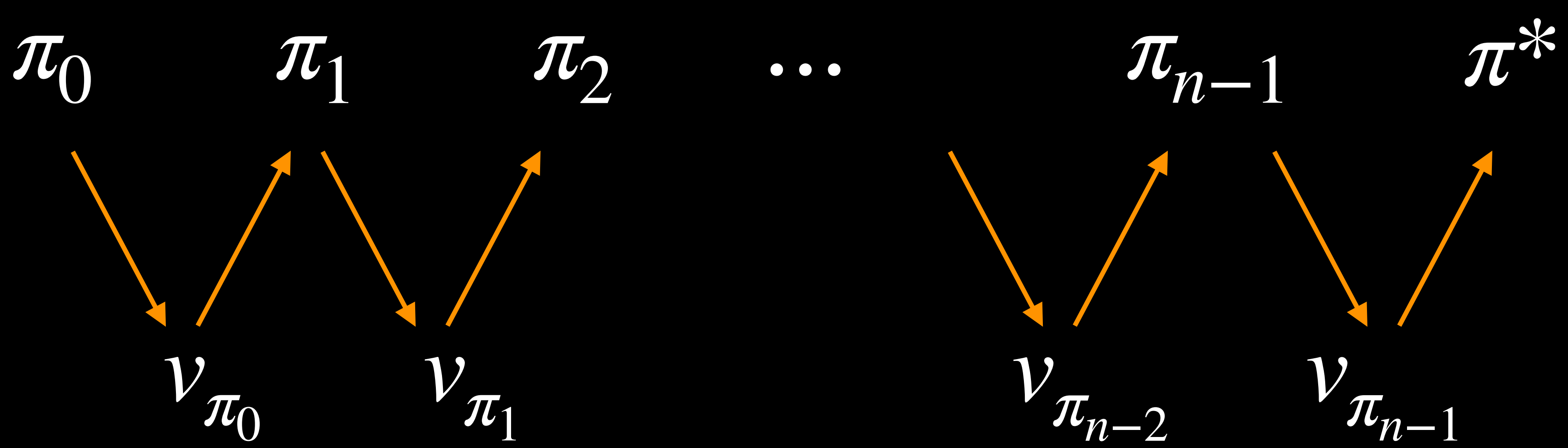

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\begin{tikzcd}
\pi_0 \arrow[dr] & & \pi_1 \arrow[dr] & & \pi_2 & \cdots \arrow[dr] & & \pi_{n-1} \arrow[dr] & & \pi^* \\
& v_{\pi_0} \arrow[ur] & & v_{\pi_1} \arrow[ur] & & & v_{\pi_{n-2}} \arrow[ur] & & v_{\pi_{n-1}} \arrow[ur] &
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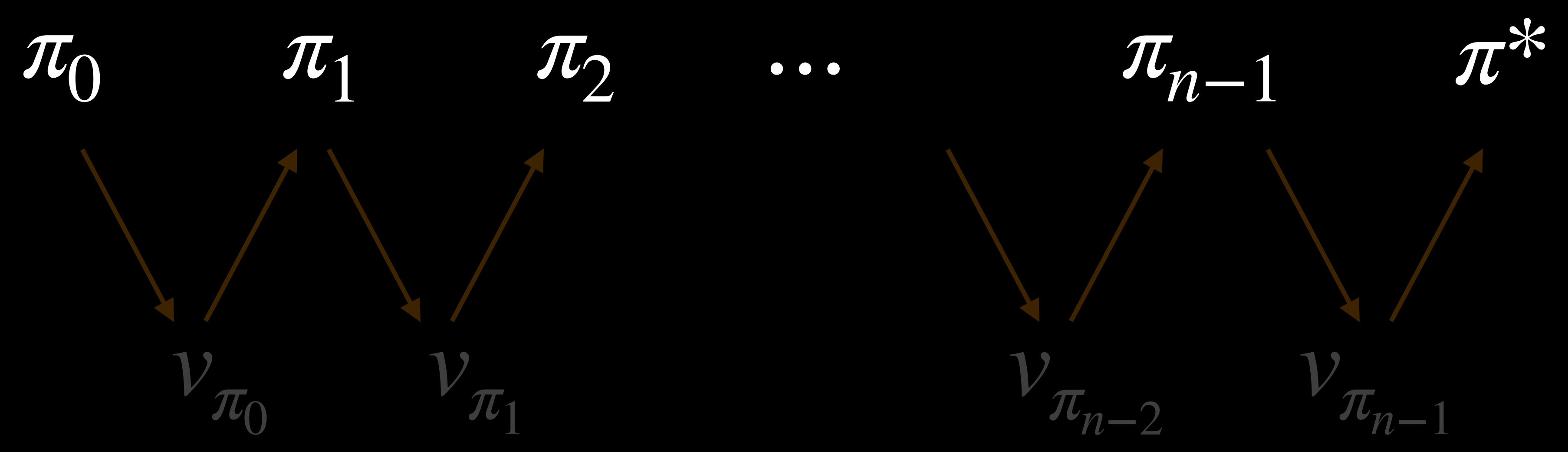

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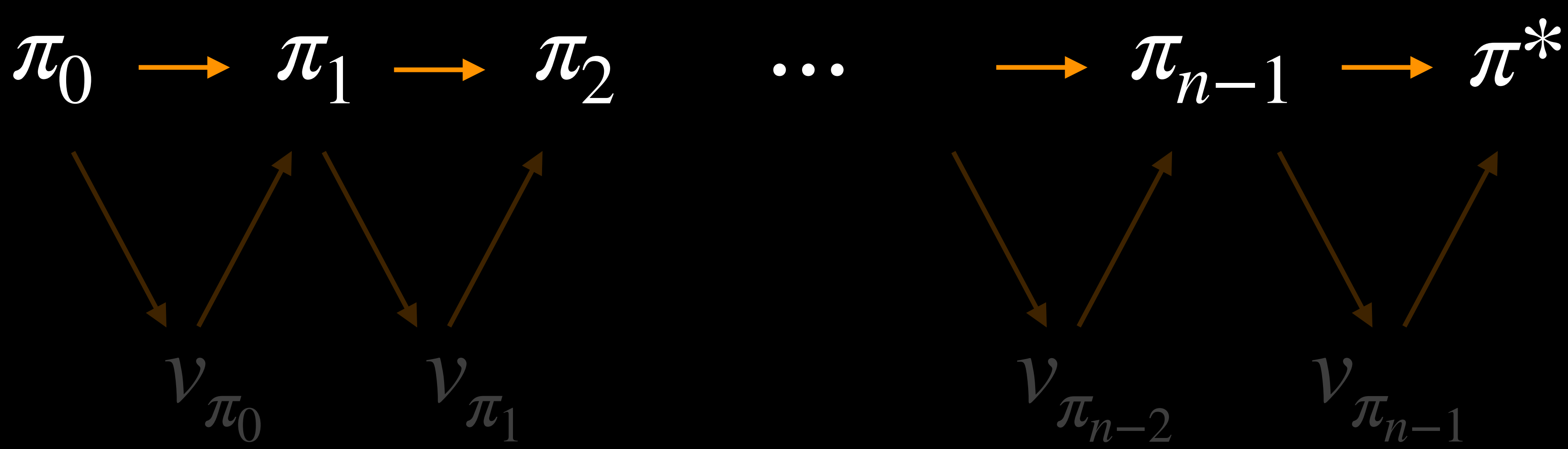

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"soft-max"

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"Gradient-Bandit Algorithm"

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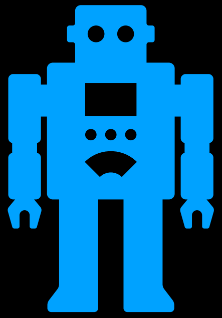

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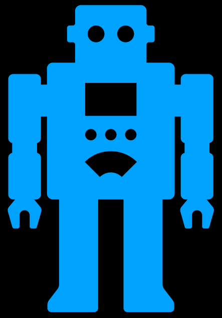

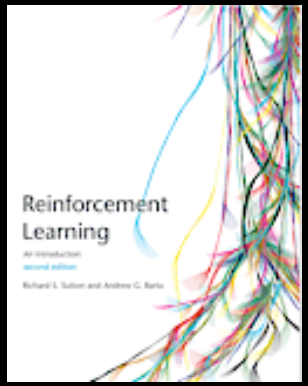

Further reading:
Sutton & Barto, 2018, *Reinforcement Learning: An Introduction*, 2nd Edition
http://incompleteideas.net/book/the-book.html

THANK YOU

Questions?

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